Math 450: Mathematics for Multimedia Midterm Examination

Name: ____________________________________

Wednesday, 6 March 2019

7 problems on 1+7 pages

Use only this test and a pen or pencil. Please write your complete answers in the space provided. You have 50 minutes.

Note: The inverse of a 2×2 matrix is

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix},$$

which exists if and only if determinant $ad - bc \neq 0$.

1. (a) Compute the sum $A2 + 1B$ (base 16). Give the answer in hexadecimal (base 16) and also in binary (base 2).

   (b) Let $x = 0.BBBBB\dots$ (base 16) be the repeating hexadecimal consisting of the repeated digit B. Write x as a rational number in decimal (base 10) and hexadecimal (base 16).

2. (a) How many integers in the set $\{0, 1, 2, 3, \ldots, 61, 62\}$ are relatively prime with 63?

   (b) Evaluate $8^{37} \pmod{63}$. (Hint: use the result from part a.)

3. Suppose that $\mathbf{x}, \mathbf{y}$ are vectors in $\mathbf{R}^{17}$, with $\|\mathbf{x}+\mathbf{y}\| = \|\mathbf{x}-\mathbf{y}\| = 5$.

   (a) Evaluate $\langle \mathbf{x}, \mathbf{y} \rangle$.

   (b) Evaluate $\|\mathbf{x}\|^2 + \|\mathbf{y}\|^2$.

   Here $\langle , \rangle$ and $\| \cdot \|$ are the usual inner product and derived norm of Euclidean space.

4. Suppose that $B = \{b_1, b_2\}$ is the basis of the inner product space $\mathbf{R}^2$ consisting of the vectors

$$b_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad b_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$

Let $B' = \{b_1', b_2'\}$ be the biorthogonal dual basis to B.

(a) Find the vectors in B'.

(b) Find scalars c_1, c_2 such that

$$\begin{pmatrix} 4 \\ 2 \end{pmatrix} = c_1 b_1 + c_2 b_2,$$

5. Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be the linear transformation defined by $T(x, y) = (2x+y, x-2y)$. Compute $\|T^2\|_{\text{op}}$ with respect to the usual Euclidean norms.

6. Compute the complex exponential Fourier series of the 1-periodic function $f(t) = \cos^2(17\pi t)$, namely, find the coefficient $c(n)$ for every integer n that gives

$$f(t) = \sum_{n=-\infty}^{\infty} c(n) e^{2\pi i n t}$$

7. Suppose that $f = f(x)$ has Fourier integral transform

$$g(\xi) \stackrel{\text{def}}{=} \int_{-\infty}^{\infty} f(x) e^{-2\pi i x \xi}\, dx.$$

Compute the Fourier integral transform of $2f(\frac{1}{3}x - 5)$ in terms of g.