

## MATH 217 – WORKSHEET 09

Q.1 Solve each of the following initial value problems using the Laplace transform.

(a)  $y' + y = e^{2x}$ ,  $y(0) = 0$ .

(b)  $y'' + 2y' + 2y = 2$ ,  $y(0) = 0$  and  $y'(0) = 1$ .

(c)  $y'' + y' = 3x^2$ ,  $y(0) = 0$  and  $y'(0) = 1$

Q.2 Use the formula  $L[y'(x)](p) = pL[y(x)](p) - y(0)$  to derive the formula

$$L \left[ \int_0^x f(t) dt \right] (p) = \frac{1}{p} L[f(x)](p).$$

Q.3 Without worrying about convergence issues, show the following:

(a)  $\int_0^\infty \frac{\sin yx}{x} dx = \frac{\pi}{2}$ , for all  $y > 0$

(b)  $\int_0^\infty \frac{\cos yx}{1+x^2} dx = \frac{\pi}{2} e^{-y}$ , for all  $y > 0$ .

Q.4 Compute the convolution  $f * g(x)$  of the following pairs of functions:

(a)  $f(x) = e^{ax}$ ,  $g(x) = e^{bx}$ .

(b)  $f(x) = e^x$ ,  $g(x) = x$ .

(c)  $f(x) = \sin at$ ,  $g(x) = \sin bt$ , where  $a^2 \neq b^2$ .

Q.5 For  $a \geq 0$ , let  $h_a$  be the unit step function with a jump at  $a$ :

$$h_a(x) = \begin{cases} 0, & x < a, \\ 1, & x \geq a. \end{cases}$$

(a) Compute the Laplace transform of  $h_a$ .

(b) Compute the convolution  $h_a * h_b(x)$  for  $a \geq 0$  and  $b \geq 0$ .

(c) How could you compute and interpret  $L[h'_a(x)]$  for  $a > 0$ ?