

### MATH 217 – WORKSHEET 06

*Q.1* For each of the following ODEs, verify that the origin is a regular singular point and find two linearly independent Frobenius series solutions.

(a)  $2xy'' + (3-x)y' - y = 0$

(b)  $2x^2y'' + xy' - (x+1)y = 0$

*Q.2* Find the Fourier series for the functions below:

(a)  $f(x) = 0$  if  $-\pi \leq x < 0$ , while  $f(x) = \sin x$  if  $0 \leq x < \pi$ .

(b)  $f(x) = 0$  if  $-\pi \leq x < 0$ , while  $f(x) = \cos x$  if  $0 \leq x < \pi$ .

*Q.3* Find the Fourier series for the functions below:

(a)  $f(x) = -1$  if  $-\pi \leq x < 0$ , while  $f(x) = +1$  if  $0 \leq x < \pi$ .

(b)  $g(x) = x - \frac{\pi}{2}$  if  $-\pi \leq x < 0$ , while  $g(x) = \frac{\pi}{2} - x$  if  $0 \leq x < \pi$ .

*Q.4* Let  $f$  be the  $2\pi$  periodic function defined on  $[-\pi, \pi)$  by

$$f(x) = \begin{cases} 0, & -\pi \leq x < 0, \\ x^2, & 0 \leq x < \pi \end{cases}$$

(a) Find the Fourier series for  $f$ .

(b) Use Dirichlet's theorem (Th.7.2.7, p.177 in the textbook) with the results from part (a) at  $x = 0$  to prove that

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} = 1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots = \frac{\pi^2}{12}$$

(c) Use Dirichlet's theorem with the part (a) series at  $x = \pi$  to prove that

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots = \frac{\pi^2}{6}$$

(d) Derive part (c) from part (b). (Hint: add  $2 \sum_n (1/[2n]^2)$  to both sides.)

*Q.5* Determine whether the following functions are odd, even, or neither.

$$x^5 \sin x, \quad x^2 \sin 2x, \quad e^x, \quad (\sin x)^3, \quad \sin x^2, \quad \cos(x+x^3), \quad \frac{\sin x}{x}, \quad x+x^2+x^3, \quad \ln \frac{1+x}{1-x}$$

*Q.6* Let  $f(x) = \pi/4$  be the constant function.

(a) Show that the sine series for  $f$  is

$$\frac{\pi}{4} = \sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \dots, \quad 0 < x < \pi.$$

(b) Let  $x = \pi/2$  in part (a) and deduce an infinite sum formula.

(c) Find the cosine series for  $f$ .