MATH 217 – WORKSHEET 04

Q.1 The ODE

$$y^{(6)} + 4y^{(5)} + 3y^{(4)} - 10y^{(3)} - 26y'' - 24y' - 8y = 0$$

has characteristic equation

$$r^6 + 4*r^5 + 3*r^4 - 10*r^3 - 26*r^2 - 24*r - 8 = (r+1)^2(r^2+2r+2)(r^2-4) = 0.$$

Find the general solution.

Q.2 Find the general solution of $y''' - 3y'' + 2y' = 10 + 18e^{3x}$

Q.3 Suppose that a straight tunnel is drilled through Mars connecting two points on the surface, which we will assume is a perfect sphere of radius $R = 3390$ km. We will also assume Mars has uniform density, and that the acceleration due to gravity is $g = 3.72$ meters per second squared at its surface.

(a) If a train coasts frictionlessly on tracks laid in the tunnel, show that the time required for a complete round trip does not depend on the locations of the surface entrances.

(b) Estimate the value of the round trip time in part (a).

(c) If the tunnel has length $2L$, where $0 < L \leq R$, what is the maximum speed attained by the train? Check your answer as $L \to 0$.

Q.4 A cylindrical buoy d meters in diameter floats with its axis vertical in water with a density of ρ kg per cubic meter. When pushed down and released, it is observed to oscillate with a period of T seconds. This all happens on a watery planet where the acceleration due to gravity is g meters per second squared at the surface.

(a) What is the formula for the mass of the buoy in kg?

(b) Use $g = 9.8$, $\rho = 1027$, $T = 1.9$, and $d = 0.6$ to compute the mass by your formula.

Q.5 The *mean distance* of a planet from its star is the semimajor axis of its elliptical orbit.

(a) Mercury's "year" (namely its orbital period) is 88 days. What is its mean distance from the Sun?

(b) Saturn's mean distance from the Sun is 9.54 times that of Earth. What is Saturn's orbital period?