MATH 217 – WORKSHEET 02

Q.1 *Bernoulli's equation* has the form

$$\frac{dy}{dx} + P(x)y = Q(x)y^n,$$

where P and Q are given functions. It is first-order linear when $n = 0$ or $n = 1$. For other values of n it can be made linear in a new dependent variable $z = z(x)$ with the change of variable $z = y^{1-n}$ and the subsequent elimination of y.

Use this method to solve the differential equation $y' + xy = xy^4$ with initial condition $y = 1$ when $x = 1$.

Q.2 Show that each of these equations is exact and find the solution.

(a) $(x + \frac{2}{y})\,dy + y\,dx = 0$

(b) $(y - x^3)\,dx + (x + y^3)\,dy = 0$

Q.3 See textbook section 2.4 for the definition of orthogonal trajectories.

(a) Find the orthogonal trajectories of the curves $y = Cx^4$.

(b) Fix an integer $n \geq 1$. Find the orthogonal trajectories of the curves $y = Cx^n$.

(c) Describe how the orthogonal trajectories in part (b) change as $n \to \infty$.

Q.4 Verify that each of the following ODEs is homogeneous and then find its general solution.

(a) $(y + xe^{y/x})\,dx - x\,dy = 0$.

(b) $x^2y' - 3xy = 2y^2$

Q.5 Solve these differential equations by finding an integrating factor.

(a) $(x + 3y^2)\,dx + 2xy\,dy = 0$

(b) $(y \ln y - 2xy)\,dx + (x + y)\,dy = 0$

Q.6 Solve these ODE initial value problems by reduction of order.

(a) $y'' = 3y'$, with $y(0) = 0$ and $y'(0) = 1$.

(b) $xy'' + y' = 2x$, with $y'(1) = 2$ and $y(1) = 0$