QUESTIONS

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Brown showed that the ring of motivic multiple zeta values is isomorphic to the vector space

$$H := \oplus_{w\in S}\mathbf{Q}\zeta(w).$$

where $S = \{w \mid w \text{ is a word of 2's and 3's }\}$ He introduce an filtration

$$H_i = \oplus_{l\leq i} \oplus_{w\in S_l} \mathbf{Q}\zeta(w).$$

where $S_l = \{w \in S \mid \text{the number of 3's is equal to } l\}$. It is know that the Lie algebra $Lie(\pi_1(MTM/\mathbf{Z})^u)$ of the unipotent part of $\pi_1(MTM/\mathbf{Z})$ is freely generated by $D_3, D_5, D_7, \dots$. As a conequence, the completed universal enveloping algebra $\mathcal{U} = \mathcal{U}(Lie(\pi_1(MTM/\mathbf{Z})))$ of $Lie(\pi_1(MTM/\mathbf{Z}))$ is isomorphic to a non-commutative formal power series ring $R\langle\langle D_3, D_5, D_7, \dots\rangle\rangle$, where $R = \mathbf{Q}[[\frac{\partial}{\partial\zeta^m(2)}]]$. Let I be the augmentation ideal

$$I := \ker(\mathcal{U} \to R).$$

By the reslut of Brown, H_i is equal to the anihilator of I^{i+1} and we have

$$H_iH_j \subset H_{i+j}$$

by the Leibniz rule. As a consequence, for $w \in S_i$ and $w' \in S_j$, we have

$$\zeta(w)\zeta(w') = \sum_{k\leq i+j, w''\in S_{i+j}} c^{w''}_{w,w'}\zeta(w'') \tag{0.1}$$

with some $c^{w''}_{w,w'}$. We can consider similar subspace H^Φ, H^Φ_i using coefficients of associators.

Problem 0.1. *Does the similar equality as (0.1) holds also for c^Φ ?*

Theorem 0.2. *If the above problem is true, then we have a splitting of the injective homomorphism*

$$\pi_1(MTM/\mathbf{Z})^u \to GT^u.$$

Proof. We will construct the homomorphism $\varphi : Lie(GT) \to Lie(\pi_1(MTM/\mathbf{Z}))$. By the assumption, the ring H^Φ becomes a sub algebra of $\mathbf{Q}[\zeta^\Phi]$. Therefore there exist right $\pi_1(MTM/\mathbf{Z}, \omega_B)$ action and left GT^{dR} action on the scheme $Spec(H)$. These two actions obviously commute to each other. By the result of Brown, the scheme $Spec(H)$ is a principal homogenous space under the right and left actons of $\pi_1(MTM/\mathbf{Z}, \omega_B)$ and $\pi_1(MTM/\mathbf{Z}, \omega_{dR})$. Therefore

Date: July 1, 2013.

the left action of an element g in GT^{dR} is induced by the unique left action of an element $\varphi(g)$ in $\pi_1(MTM/\mathbf{Z}, \omega_{dR})$. We can check that φ defines a homomorphism of algebraic group $GT^{dR} \to \pi_1(MTM/\mathbf{Z}, \omega_{dR})$. $\square$