Math 132: Discussion Session: Week 3

Directions: In groups of 3-4 students, work the problems on the following page. Below, list the members of your group and your answers to the specified questions. Turn **this paper** in at the end of class. You do not need to turn in the question page or your work.

Additional Instructions: We'll spend some of the time on this worksheet, and some of the time reviewing for the exam. It is okay if you do not completely finish all of the problems. Also, each group member should work through each problem, as similar problems may appear on the exam.

Scoring:

Correct answers	Grade
0–1	0%
2–3	80%
4–5	100%

Group Members:

5.3: Fundamental Theorem of Calculus.

(1) (a) $F'(x) =$

(b) $G'(x) =$

(2) $F(x) =$

(3) (a) State the critical point(s) and whether F has a local max, local min, or neither at each one:

(b) State the inflection point(s) and how the concavity of F changes at each one:

Math 132 Discussion Session: Week 2

5.3: Fundamental Theorem of Calculus.

(1) Using the Fundamental Theorem of Calculus, compute the derivatives of the following functions:

(a) $F(x) = \displaystyle\int_2^{e^{x^2}} \frac{x+1}{x-1}\, dx$

Solution: Let $H(x)$ be an antiderivative of $\frac{x+1}{x-1}$, so $H'(x) = \frac{x+1}{x-1}$. Then

$$\int_2^{e^{x^2}} \frac{x+1}{x-1}\, dx = H(e^{x^2}) - H(2),$$

$$\frac{d}{dx}\int_2^{e^{x^2}} \frac{x+1}{x-1}\, dx = H'(e^{x^2}) \cdot \left(e^{x^2}\right)(2x) - 0$$

$$= \boxed{\frac{e^{x^2}+1}{e^{x^2}-1} \cdot \left(2xe^{x^2}\right).}$$

(b) $G(x) = \displaystyle\int_{\cos x}^{x^2} \ln(x+3)\, dx.$

Solution: Let $K(x)$ be an antiderivative of $\ln(x+3)$, so $K'(x) = \ln(x+3)$. Then

$$\int_{\cos x}^{x^2} \ln(x+3) = K(x^2) - K(\cos x),$$

$$\frac{d}{dx}\int_{\cos x}^{x^2} \ln(x+3) = K'(x^2)\cdot(2x) - K'(\cos x)\cdot(-\sin x)$$

$$= \boxed{\ln(x^2+3)\cdot(2x) + \ln(\cos x + 3)\sin x.}$$

(2) Using the Fundamental Theorem of Calculus, give an antiderivative $F(x)$ of $f(x) = \sin^2(x) + e^{x^2}$ satisfying $F(3) = 0$. Your answer can involve a definite integral.

Solution: According to the Fundamental Theorem of Calculus,

$$\frac{d}{dx}\int_a^x \left(\sin^2(t) + e^{t^2}\right)\, dt = \sin^2(x) + e^{x^2}.$$

Thus, any function of the form

$$F(x) = \int_a^x \left(\sin^2(t) + e^{t^2}\right)\, dt$$

is an antiderivative of $\sin^2(x) + e^{x^2}$. However, we also need our antiderivative to satisfy $F(3) = 0$. Plugging that in, we find that

$$0 = \int_a^3 \left(\sin^2(t) + e^{t^2}\right)\, dt.$$

One easy way to accomplish that is by setting $a = 3$, since $\int_3^3 f(t)\, dt = 0$. Thus, our antiderivative is

$$F(x) = \boxed{\int_3^x \left(\sin^2(t) + e^{t^2}\right)\, dt} = \boxed{\int_3^x f(t)\, dt}.$$

(3) Let $F(x) = \displaystyle\int_0^x (t^2 - 6t + 8)\, dt.$

(a) Find the critical points of F (i.e. the points where $F'(x) = 0$) and determine whether they are local minima or local maxima.

Solution: We could take an antiderivative to compute $F(x)$, but that would be silly since the next step is to take a derivative.

The Fundamental Theorem of Calculus tells us that

$$F'(x) = \frac{d}{dx}\int_0^x (t^2 - 6t + 8)\,dt = x^2 - 6x + 8 = (x-2)(x-4).$$

Thus, the critical points of $F(x)$ occur when

$$0 = (x-2)(x-4),$$

that is, when $x = 2$ or $x = 4$.

Investigating the expression $F'(x) = (x-2)(x-4)$ further, we see that $F'(x) > 0$ when $x < 2$ or $x > 4$, and $F'(x) < 0$ when $2 < x < 4$. Therefore, the function $F(x)$ is increasing until we get to $x = 2$, at which point it starts decreasing, so $\boxed{F(x) \text{ has a local maximum at } x = 2.}$ After that, the function is decreasing until $x = 4$, at which point it starts increasing, so $\boxed{F(x) \text{ has a local minimum at } x = 4.}$

(b) Find the points of inflection of F (i.e. the points where $F''(x) = 0$) and determine whether the concavity changes from up to down or from down to up at each one.

Solution: We already computed $F'(x)$, so, using the product rule, we compute that

$$F''(x) = \frac{d}{dx}((x-2)(x-4)) = 1 \cdot (x-4) + (x-2) \cdot 1 = 2x - 6.$$

Solving, we find that $\boxed{F''(x) = 0 \text{ when } x = 3.}$

Looking at the expression $2x - 6$, when $x < 3$, we see that $F''(x) < 0$, so F is concave down. When $x > 3$, we see that $F''(x) > 0$, so F is concave up. Thus, at the inflection point at $x = 3$, $\boxed{F \text{ changes from being concave down to being concave up.}}$