An introduction to pricing methods for credit derivatives

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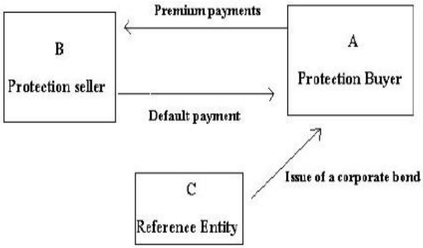
Credit derivatives

1. What are they?
   - Securities used to manage and trade "credit risk" of firm; e.g. the risk that the firm will default in a debt contract.
   - The payoff of a credit derivative depends on the occurrence of a credit event affecting a financial entity; e.g. Payoff triggered by a default event.
2. Two important examples: Credit Default Swaps (CDS) and Collaterized Debt Obligations (CDO).
3. Credit Default Swaps:
   - The most important derivative (2002: accounting for about 67% of the credit derivatives market).
   - Over-the-counter market for CDS written on large corporations is fairly liquid.
   - Natural underlying security for more complex credit derivatives.
   - CDS quotes data are used to calibrate pricing methods.

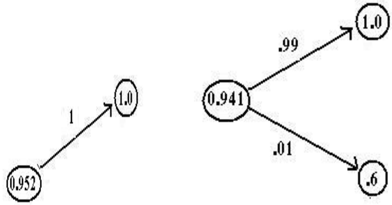
Credit Default Swaps

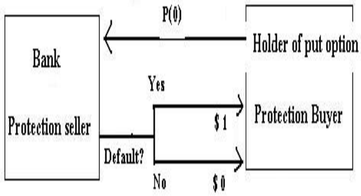
1. Three entities: A protection buyer, B protection seller, and C the reference entity.
2. A contract where A pays periodic premium payments until maturity or until the default of C;
3. If C defaults, B pays A a default payment (for instance, default payment could mimic the loss that A suffers on a bond issue by C to A).
4. The premium payments are quoted in annualized percentage x^* of the notional value of the reference asset. This rate x^* is called the CDS spread.



A simple example

1. Set-up:
   - Zero-coupon bond exposed to default, with maturity $T = 1$
   - Recovery rate $1 - \delta = 60\%$
   - Objective default probability $p = 1\%$
   - Current price of defaultable bond is .941.
   - A “risk-free" default-free zero-coupon bond with interest rate 5%.
   - Current price of default-free bond is $(1.05)^{-1} = .952$.
2. Default put option:
   - This contract pays 1 dollar if the bond defaults and pays 0 otherwise.
   - This can be thought as a simplified CDS with only one premium





Pricing and hedging of the Default Put Option

1. Fundamental problems of finance:
   - What is the fair price of the credit derivative?
   - Is it possible to replicate the payoff of the derivative trading on the underlyings?
2. Answers:
   - Replicating portfolio:
     - Go short 2.5 dollars in the defaultable bond
     - Put $50/21 \approx 2.38$ dollars in the default-free bond
     - Time $t = 1$ value of the portfolio

$$V_1 = \begin{cases} (-2.5)(1) + (\frac{50}{21})(1.05) = 0 & \text{if no default} \\ (-2.5)(.6) + (\frac{50}{21})(1.05) = 1 & \text{if default} \end{cases}$$

     - Time $t = 0$ initial endowments for portfolio:

$$V_0 = (-2.5)(.941) + \frac{50}{21} \approx .0285.$$

   - Arbitrage-free price of the option: $P(0) = .0285$.

Martingale or Risk-neutral valuation

Fundamental question:

Is the price consistent with the “martingale method"?

Martingale Method:

1. Calibration: Determine a measure Q so that all traded assets are martingales:

$$\text{Initial market price} = E^Q\left\{\frac{\text{Value at expiration}}{\text{Value of \$1 at expiration}}\right\}$$

$$.941 = \frac{0.6}{1.05}\cdot q + \frac{1}{1.05}\cdot(1-q) \quad\Longrightarrow\quad q = .03$$

2. Risk-neutral pricing:

$$P(0) = E^Q\{\text{Discounted payoff}\} \Longrightarrow P(0) = \frac{1}{1.05}.03 + \frac{0}{1.05}\cdot .97.$$

Reduced-form model for CDS

1. Fundamental assumption: The objects of interest are directly modeled under the risk-neutral probability measure Q.
2. Objects of interest:
   - A (deterministic) risk-free (default-free) bond market with short-interest rate $r(t)$; that is, the time 0 price of a ZCB with maturity t is

$$P_0(0; t) = e^{-\int_0^t r(s)ds}.$$

   - δ=Recovery rate if default (deterministic).
   - τ=random default time of the reference entity.
3. Specification of the CDS:
   - Payments of premium are due at $0 < t_1 < \cdots < t_N = T$.
   - If $t_k < \tau$, protection buyer (A) pays premium $x^*(t_k - t_{k-1})$ to protection seller (B) at time t_k. x^* is called swap spread.
   - If $\tau \leq T$, then B makes the default payment δ to A at times τ.
   - x^* is set so that the value of the CDS contract at time $t = 0$ is equal to 0.

Pricing of CDS

1. Pricing:
   - Value of the premium payment leg:

$$V^{prem}(x) = \sum_{k=1}^{N} p_0(0, t_k)(t_k - t_{k-1})Q(\tau > t_k).$$

   - Value of the default payment leg:

$$V^{def} = \delta \int_0^{t_N} f_\tau(t) e^{-\int_0^t r(s)ds} dt,$$

     where f_τ is the density of τ under Q.
   - The fair (arbitrage-free) CDS spread x^* is such that

$$V^{prem}(x^*) = V^{def}.$$

2. A common model: Default time τ is given in terms of its "hazard rate" $\gamma^Q(t)$:

$$P(\tau > t) = e^{-\int_0^t \gamma^Q(t)dt} \Longleftrightarrow \lim_{\delta \to 0} \frac{1}{\delta} Q(\tau \le t + \delta | \tau > t) = \gamma^Q(t).$$

What about multi-name credit derivatives?

1. Suppose that we are interested in trading and managing the credit risk of several firms.
2. An example:
   - Say that we bear a portfolio consists of m corporate bonds issued by firms with corresponding default times $\tau_1, \ldots, \tau_m$.
   - Our “exposure" to firm i is e_i.
   - The recovery rate if firm i defaults is δ_i.
   - The total loss at time t will be

$$L_t = \sum_{i=1}^{m} \delta_i e_i \mathbf{1}_{\{\tau_i \leq t\}}.$$

   - What will be the distribution of the total loss if the firms are correlated?
   - How to model the dependence between the default times?

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