Math 2200 Spring 2015, Exam 3

You may use *any* calculator. You may use a 4×6 inch notecard as a cheat sheet.

Warning to the Reader! If you are a student for whom this document is a historical artifact, be aware that the definitions and conventions on which some of the questions on this exam are based may differ from those adopted in your course. This will likely be the case for the degrees of freedom used in the construction of a two-sample confidence interval.

1. In this problem and the one that follows, X is the number of dots that are face-up when a fairly-balanced, 6-sided, *non-standard* die is rolled. The die is nonstandard because, for each $k = 1, 2, 3$, the number of faces with k dots is k (and there are no faces with 4, 5, and 6). What is Var(X)?

A) $\frac{5}{12}$ B) $\frac{4}{9}$ C) $\frac{17}{36}$ D) $\frac{1}{2}$ E) $\frac{19}{36}$
F) $\frac{5}{9}$ G) $\frac{7}{12}$ H) $\frac{11}{18}$ I) $\frac{23}{36}$ J) $\frac{2}{3}$

Solution. *The set of values X may assume is* $\{1,2,3\}$. *If* f *is the probability function of X, then* $f(1) = 1/6$, $f(2) = 2/6 = 1/3$, *and* $f(3) = 3/6 = 1/2$. *Therefore,*

$$E(X) = 1 \times \frac{1}{6} + 2 \times \frac{1}{3} + 3 \times \frac{1}{2} = \frac{7}{3}$$

and

$$Var(X) = E\left(X^2\right) - \left(E\left(X\right)\right)^2 = 1^2 \times \frac{1}{6} + 2^2 \times \frac{1}{3} + 3^2 \times \frac{1}{2} - \left(\frac{7}{3}\right)^2 = \frac{5}{9}.$$

Answer: ***F***

2. The die of the preceding problem is rolled 5 times, with results X_1, X_2, X_3, X_4, X_5 that are independent and identically distributed. What is the standard deviation of the sample mean?

A) 0.0237 B) 0.0624 C) 0.1011 D) 0.1398 E) 0.1785
F) 0.2172 G) 0.2559 H) 0.2946 I) 0.3333 J) 0.3720

Solution. *The variance of the sample mean is*

$$\begin{aligned}
Var\left(\frac{X_1 + X_2 + X_3 + X_4 + X_5}{5}\right) &= \frac{1}{5^2}\, Var\left(X_1 + X_2 + X_3 + X_4 + X_5\right) \\
&= \frac{1}{25}\left(Var\left(X_1\right) + Var\left(X_2\right) + Var\left(X_3\right) + Var\left(X_4\right) + Var\left(X_5\right)\right) \\
&= \frac{1}{25} \times 5 \times Var\left(X_1\right) \\
&= \frac{1}{5} \times \frac{5}{9} \\
&= \frac{1}{9}.
\end{aligned}$$

It follows that the standard deviation is $\sqrt{1/9}$, *or 1/3, 0r 0.3333.*

Answer: ***I***

3. If X, Y, and W are independent random variables with variances 5, 3, and 0.2 respectively, then what is the standard deviation of $X/2 - 4\,Y/3 + 3\,W$?

A) 2.4181 B) 2.8954 C) 3.3727 D) 3.8500 E) 4.3273
F) 4.8046 G) 5.2819 H) 5.7592 I) 6.2365 J) 6.7138

Solution. *We have Var(X/2) = Var(X)/4 = 5/4, Var(-4Y/3) = (16/9)Var(Y) = 16/3, and Var(3W) = 9Var(W) = 1.8. Therefore*

$$Sd\left(\frac{1}{2}\,X-\frac{4}{3}\,Y+3\,W\right)=\sqrt{Var\left(\frac{1}{2}\,X-\frac{4}{3}\,Y+3\,W\right)}=\sqrt{\frac{5}{4}+\frac{16}{3}+1.8}=\sqrt{8.3833333}=2.8954.$$

Answer: ***B***

4. Trans fat is a poison that is currently permitted as a food additive in the U.S. Furthermore, the U.S. Food and Drug Administration has the following rounding rule, reproduced here *verbatim*, for declaring the weight of trans fat on nutrition labels: " < .5 g – express as 0." (Thus, 0.49 g is not rounded to 0.5 g but to 0 g.) Suppose that one unit of a food product has mean trans fat content 0.45 g with standard deviation 0.024 g. If 31 units are consumed, then the total amount of tans fat consumed is not 31×0 g, as the labels imply. What, approximately, is the probability that at least 14 grams (about half an ounce) of trans fat are consumed? Assume that the amounts of trans fat in different units are independent.

   A) 0.0607 B) 0.1096 C) 0.1585 D) 0.2074 E) 0.2563
   F) 0.3052 G) 0.3541 H) 0.4030 I) 0.4519 J) 0.5008

Solution. *Let X be the amount of trans fat in a unit of the food. Then*

$$\begin{aligned}
P(X_1+\cdots+X_{31}>14) &= P\left(\frac{X_1+\cdots+X_{31}}{31}>frac1431\right)\\
&= P\left(\overline{X}>0.4516129\right)\\
&= P\left(\frac{\overline{X}-0.45}{0.024/\sqrt{31}}>\frac{0.4516129-0.45}{0.024/\sqrt{31}}\right)\\
&\approx P(Z>0.374177)\\
&= 1-\Phi(0.374177)\\
&= 1-0.6458637\\
&= 0.3541363.
\end{aligned}$$

Answer: ***G***

5. Yearly total rainfall X in Los Angeles is a normal random variable with mean 12.08 inches and standard deviation 3.1 inches. Assume that the values of X in different years are independent. Assume that climate change in the next few years will not affect the distribution of X. What is the probability that the total rainfall in Los Angeles in 2017 will exceed the total rainfall in Los Angeles in 2016 by 4 inches?

   A) 0.0527 B) 0.0710 C) 0.0893 D) 0.1076 E) 0.1259
   F) 0.1442 G) 0.1625 H) 0.1808 I) 0.1991 J) 0.2174

Solution. *Let X_1 and X_2 be the yearly rainfall totals for Los Angeles in 2016 and 2017 respectively. Observe that*

$$Sd(X_2+(-X_1))=\sqrt{\left(Sd(X_2)\right)^2+\left(Sd(-X_1)\right)^2}=\sqrt{(3.1)^2+(3.1)^2}=4.384062.$$

It follows that

$$\begin{aligned}
P(X_2 > X_1 + 4) &= P(X_2 + (-X_1) > 4) \\
&= P\left(\frac{X_2 + (-X_1) - \big(12.08 + (-12.08)\big)}{4.384062} > \frac{4}{4.384062}\right) \\
&= P(Z > 0.91239586) \\
&= 1 - \Phi(0.91239586) \\
&= 1 - 0.81921982 \\
&= 0.18078018.
\end{aligned}$$

Answer: ***H***

6. Suppose that $\mathrm{X}_1, \mathrm{X}_2, \mathrm{X}_3, Y$ are i.i.d. with normal distribution that has mean 1 and standard deviation 3. What is $\mathrm{P}\,(2\,\mathrm{Y} > \mathrm{X}_1 + \mathrm{X}_2 + \mathrm{X}_3)$?

A) 0.0872	B) 0.1275	C) 0.1678	D) 0.2081	E) 0.2484
F) 0.2887	G) 0.3290	H) 0.3693	I) 0.4096	J) 0.4499

Solution. *Let $W = 2\,Y - X_1 - X_2 - X_3$. Then W is normal with mean -1 and standard deviation $\sqrt{4\,(3)^2 + (3)^2 + (3)^2 + (3)^2}$, or 7.937253933. Thus,*

$$\begin{aligned}
P(2\,Y > X_1 + X_2 + X_3) &= P(W > 0) \\
&= P(W - (-1) > 0 - (-1)) \\
&= P\left(\frac{W - (-1)}{7.937253933} > \frac{1}{7.937253933}\right) \\
&= P\left(Z > \frac{1}{7.937253933}\right) \\
&= 1 - \Phi(0.12598816) \\
&= 1 - 0.55012935 \\
&= 0.44987065.
\end{aligned}$$

Answer: ***J***

7. Suppose that X, Y, and Z are independent standard normal random variables, what is $\mathrm{P}\left(\sqrt{\mathrm{X}^2 + \mathrm{Y}^2} \leq \mathrm{Z}\right)$?

A) 0.1464	B) 0.1893	C) 0.2322	D) 0.2751	E) 0.3180
F) 0.3609	G) 0.4038	H) 0.4467	I) 0.4896	J) 0.5325

Solution. *First, rewriting the probability as $P\left(0 \leq Z^2 - X^2 - Y^2\right)$ is not unreasonable, but it doesn't help: we have no rule to handle such a linear combination of chi-squared random variables. To answer this question, we use the definition of the Student-t distribution with 2 degrees of freedom. To wit, if Z is standard normal, if Y is chi-squared with 2 degrees of freedom, and if Z and Y are independent, then $Z/\sqrt{Y/2}$ is Student-t with*

2 degrees of freedom. Thus,

$$\begin{aligned} P\left(\sqrt{X^2+Y^2} \le Z\right) &= P\left(1 \le \frac{Z}{\sqrt{X^2+Y^2}}\right) \\ &= P\left(\sqrt{2} \le \frac{Z}{\sqrt{\left(X^2+Y^2\right)/2}}\right) \\ &= P(1.4142136 \le t_2) \\ &= 0.14644661. \end{aligned}$$

Note: To answer this problem using inverse look-up from a table, then there was no need for interpolation. Iin the row for t_2, the value 1.4142136 lies between two tabulated values: 1.3862 and 1.8856. The required probability therefore lies between the corresponding probabilities, 0.150 and 0.100. That is, the answer is less than 0.150 (and bigger than 0.100, not that that mattered here). Only one of the offered answer choices was smaller than 0.150.

Answer: ***A***

8. Births at a hospital are a Poisson process with an average of 1.6 per hour. What is the probability that there will be at least 3 births in a two hour period?

A) 0.48	B) 0.50	C) 0.52	D) 0.54	E) 0.56
F) 0.58	G) 0.60	H) 0.62	I) 0.64	J) 0.66

Solution. *The number X of births in a two hour period has Poisson distribution with parameter $\lambda = 3.2$. The required probability is*

$$1 - \left(e^{-\lambda}\frac{\lambda^0}{0!} + e^{-\lambda}\frac{\lambda^1}{1!} + e^{-\lambda}\frac{\lambda^2}{2!}\right), \text{ or } 0.62.$$

Answer: ***H***

9. The number X of days per month that a machine breaks down and requires repairs has a Poisson distribution with mean and variance both equal to 3. (The variance of a Poisson distribution is always equal to its mean.) The monthly cost of maintenance, including both routine servicing and breakdown repair, is $128 + 33\,\mathrm{X}^2$. What is the expected monthly cost of maintenance? (Use the given values of both the mean and variance: a direct calculation involving X^2 requires mathematics beyond the level of this course.)

A) 500	B) 504	C) 508	D) 512	E) 516
F) 520	G) 524	H) 528	I) 532	J) 536

Solution. *The first step is to calculate $E\left(X^2\right)$. To do so, we use the identity $Var(X) = E\left(X^2\right) - \left(E(X)\right)^2$, which we rewrite as $E\left(X^2\right) = Var(X) + \left(E(X)\right)^2$. Therefore, $E\left(X^2\right) = 3 + (3)^2 = 12$. Thus*

$$E\left(128 + 33\,X^2\right) = 128 + 33\,E\left(X^2\right) = 128 + 33 \times 12 = 524.$$

Answer: ***G***

10. In a certain region, the number of daily tremors that exceed a certain magnitude is a Poisson random variable with mean 2.4. Assuming that the numbers of tremors on different days are independent, approximate the probability that there are more than 75 tremors in a 31 day month. Use the normal

approximation with correction for continuity.

A) 0.4250 B) 0.4331 C) 0.4412 D) 0.4493 E) 0.4574
F) 0.4655 G) 0.4736 H) 0.4817 I) 0.4898 J) 0.4979

Solution. *Let X_j be the number of tremors on the j^{th} day. From the information of the preceding problem, we calculate the mean and variance of $X_1 + \cdots + X_{31}$ to be 31×2.4, or 74.4. Thus,*

$$\begin{aligned} P(X_1 + \cdots + X_{31} > 75) &= P(X_1 + \cdots + X_{31} \geq 76) \\ &\approx P\left(N\left(74.4, \sqrt{74.4}\right) \geq 75.5\right) \\ &= P\left(\frac{N(74.4, \sqrt{74.4}) - 74.4}{\sqrt{74.4}} \geq \frac{75.5 - 74.4}{\sqrt{74.4}}\right) \\ &= P(Z \geq 0.1275281963) \\ &= 1 - \Phi(0.1275281963) \\ &= 0.4492612. \end{aligned}$$

The R code `1-sum(dpois(0:75, 31*2.4))` *evaluates the exact value as 0.441755. Without the correction for continuity, the normal approximation results in the somewhat poor approximation 0.4264202. (To obtain this value, use 76 instead of 75.5 in the calculation just given.)*

Answer: ***D***

11. According to a recent report, the proportion of American adults who have a cholesterol level above 200 is 0.52. In a random sample of 100 adult Americans, what, approximately, is the probability that between 40 and 60 of them have cholesterol levels above 200? Do not use the correction for continuity in this problem: you will be asked to use it in the next. To be clear: the number of adult Americans to which the requested probability refers is at least 40 but no more than 60.

A) 0.9328 B) 0.9339 C) 0.9350 D) 0.9361 E) 0.9372
F) 0.9383 G) 0.9394 H) 0.9405 I) 0.9416 J) 0.9427

Solution. *The exact probability is $\sum_{k=40}^{60} \binom{100}{k} (0.52)^k (0.48)^{100-k}$, or 0.9499513. Let $X_j = 1$ if the j^{th} samplee has cholesterol level above 200, and let $X_j = 0$ otherwise. The normal approximation without the correction for continuity is*

$$\begin{aligned} P(40 \leq X_1 + \cdots + X_{100} \leq 60) &\approx P\left(40 \leq N\left(100 \times 0.52, \sqrt{100 \times 0.52 \times 0.48}\right) \leq 60\right) \\ &= P\left(\frac{40 - 52}{\sqrt{100 \times 0.52 \times 0.48}} \leq \frac{N\left(100 \times 0.52, \sqrt{100 \times 0.52 \times 0.48} - 52\right)}{\sqrt{100 \times 0.52 \times 0.48}} \leq \frac{(60 - 52)}{\sqrt{100 \times 0.52 \times 0.48}}\right) \\ &= P(-2.401922306 \leq Z \leq 1.601281538) \\ &= \Phi(1.601281538) - \Phi(-2.401922306) \\ &= 0.9371881. \end{aligned}$$

The above evaluations were obtained in R. Using a table to calculate $\Phi(1.601281538) - \Phi(-2.401922306)$, or $\Phi(1.601281538) - 1 + \Phi(2.401922306)$, we obtain

$$0.9452 + \frac{(1.601281538 - 1.6)}{(1.61 - 1.60)}(0.9463 - 0.9452) - 1 + 0.9918 + \frac{(2.401922306 - 2.4)}{(2.41 - 2.40)}(0.9920 - 0.9918),$$

which simplifies to 0.9371794153. Each way, the answer rounds to 0.9372.

Answer: ***E***

12. As stated in the preceding problem, the proportion of American adults who have a cholesterol level above 200 is 0.52. In a random sample of 100 adult Americans, what, approximately, is the probability that between 40 and 60 of them have cholesterol levels above 200? To be clear: at least 40 but no more than 60. Use the correction for continuity in this problem.

A) 0.9439	B) 0.9450	C) 0.9461	D) 0.9472	E) 0.9483
F) 0.9494	G) 0.9505	H) 0.9516	I) 0.9527	J) 0.9538

Solution. *The normal approximation is* $X_1 + \cdots + X_{100} \approx N\big(100 \times 0.52, \sqrt{100 \times 0.52 \times 0.48}\big)$*, or* $X_1 + \cdots + X_{100} \approx N(52, 4.996)$*. Thus, using the correction for continuity, we have*

$$\begin{aligned} P(39.5 \le X_1 + \cdots + X_{100} \le 60.5) &\approx P(39.5 \le N(52, 4.996) \le 60.5) \\ &= P\left(\frac{39.5 - 52}{4.996} \le \frac{N(52, 4.996) - 52}{4.996} \le \frac{60.5 - 52}{4.996}\right) \\ &= P(-2.502002402 \le Z \le 1.701361634) \\ &= \Phi(1.701361634) - \Phi(-2.502002402) \\ &= 0.9493878. \end{aligned}$$

The above evaluations were obtained in R. Using a table to calculate $\Phi(1.701361634) - \Phi(-2.502002402)$*, or* $\Phi(1.701361634) - 1 + \Phi(2.502002402)$*, we obtain*

$$0.9554 + \frac{(1.701361634 - 1.7)}{(1.71 - 1.70)}(0.9564 - 0.9554) - 1 + 0.9938 + \frac{(2.502002402 - 2.5)}{(2.51 - 2.50)}(0.9940 - 0.9938),$$

which simplifies to 0.9493762114. Each way, the answer rounds to 0.9494.
Answer: ***F***

13. The cholesterol level of American adults was recently reported to be normally distributed with mean 202 and standard deviation 41. In a random sample of size 100, what is the probability that the average cholesterol level is between 200 and 210?

A) 0.62	B) 0.64	C) 0.66	D) 0.68	E) 0.70
F) 0.72	G) 0.74	H) 0.76	I) 0.78	J) 0.80

Solution. *We calculate*

$$\begin{aligned} P\big(200 \le \overline{X} \le 210\big) &= P\left(\frac{(200 - 202)}{41/sqrt100} \le \frac{\overline{X} - 202}{41/sqrt100} \le \frac{(210 - 202)}{41/sqrt100}\right) \\ &= P(-0.4878048780 \le Z \le 1.951219512) \\ &= \Phi(1.951219512) - \Phi(-0.4878048780) \\ &= 0.6616405. \end{aligned}$$

Answer: ***C***

14. As stated in the preceding problem, the cholesterol level of American adults was recently reported to be normally distributed with mean 202 and standard deviation 41. If a random sample of size 18 is drawn from the population, what is the probability that the the sample variance is greater than $(34.447)^2$?

A) 0.400	B) 0.500	C) 0.600	D) 0.700	E) 0.750
F) 0.800	G) 0.900	H) 0.950	I) 0.975	J) 0.990

Solution. *Let* S^2 *be the sample variance. Then* $\dfrac{(18-1)}{(41)^2}\cdot S^2 \sim \chi^2_{18-1}$. *We have*

$$\begin{aligned} P\left(S^2 > (34.447)^2\right) &= P\left(\frac{(18-1)}{(41)^2}\cdot S^2 > \frac{(18-1)}{(41)^2}\cdot(34.447)^2\right) \\ &= P\left(\chi^2_{17} > 12.00008\right) \\ &= 0.8001324. \end{aligned}$$

Answer: ***F***

15. The cholesterol levels of a large *subpopulation* of American adults is $\mathrm{N}(\mu, \sigma)$ with unknown subpopulation mean μ and unknown subpopulation standard deviation σ. The cholesterol levels of 4 randomly chosen members of the subpopulation are 194, 201, 204, and 205. Based on this sample, what is the probability that the subpopulation mean μ is greater than 199.95?

A) 0.50 B) 0.55 C) 0.60 D) 0.65 E) 0.70
F) 0.75 G) 0.80 H) 0.85 I) 0.90 J) 0.95

Solution. *Let* X_1, X_2, X_3, X_4 *be the four cholesterol levels. The observed sample mean is 201, the sample variance is 24.66667, and the sample standard deviation is* 4.966555. *Furthermore* $\left(\overline{X}-\mu\right)/(S/\sqrt{4}) \sim t_{4-1}$. *Thus,*

$$\begin{aligned} P(\mu > 199.95) &= P(-\mu < -199.95) \\ &= P(\overline{X}-\mu < \overline{X}-199.95) \\ &= P\left(\frac{\overline{X}-\mu}{S/\sqrt{4}} < \frac{\overline{X}-199.95}{S/\sqrt{4}}\right) \\ &= P\left(t_3 < \frac{\overline{X}-199.95}{S/\sqrt{4}}\right) \\ &= P\left(t_3 < \frac{201-199.95}{4.966555/\sqrt{4}}\right) \\ &= P\left(t_3 < 0.4228282985\right) \\ &= 1 - P\left(t_3 > 0.4228282985\right) \\ &= 0.6495506. \end{aligned}$$

Answer: ***D***

16. According to a Yankelovich Partners poll of 1000 adult Americans, 45% confessed to believing in faith healing (as reported by USA Today, 20 April 1998). Based on this survey, what was the upper bound of a 95% confidence interval for the proportion of the population who believe in faith healing?

A) 0.4582 B) 0.4695 C) 0.4808 D) 0.4921 E) 0.5034
F) 0.5147 G) 0.5260 H) 0.5373 I) 0.5486 J) 0.5599

Solution. *The requested upper bound is*

$$0.45 + z_{0.05/2}\sqrt{\frac{(0.45)(0.55)}{1000}} = 0.45 + 1.959964 \cdot 0.01573213 = 0.4808344.$$

Answer: ***C***

17. Tweedledum and Tweedledee agreed to have a political battle. Based on a poll of size 252, the interval $[0.3600677, 0.5052792]$ was determined to be a $100(1-\alpha)\%$ confidence interval for the proportion of the population who supported Tweedledum. What was the confidence level $100(1-\alpha)\%$?

A) 90% B) 91% C) 92% D) 93% E) 94%
F) 95% G) 96% H) 97% I) 98% J) 99%

Solution. *First we obtain* $\widehat{p} = (0.5052792 + 0.3600677)/2 = 0.4326735$. *Next we obtain* $ME(\widehat{p}) = 0.5052792 - 0.4326735 = 0.0726057$. *From our determination of* $\widehat{p}$ *and the given value of* n, *we can also find the standard error:*

$$SE(\widehat{p}) = \sqrt{0.4326735(1-0.4326735)/252} = 0.03121019.$$

From our calculations of $ME(\widehat{p})$ *and* $SE(\widehat{p})$, *we can find the critical value*

$$z_{\alpha/2} = \frac{ME(\widehat{p})}{SE(\widehat{p})} = \frac{0.0726057}{0.03121019} = 2.326346.$$

Next we observe that $\Phi(2.326346) = 0.99 = 1 - 0.01$. *Thus,* $\alpha/2 = 0.01$, *or* $\alpha = 0.02$. *The interval is a 98% confidence interval.*

Answer: ***I***

18. A seed company desires to assess potential sales before it embarks on a relatively costly program to develop a line of herbicide-tolerant seeds. Farmers will be surveyed to find the proportion willing to incur the higher seed costs. In order to have a margin of error no greater than 0.04 and a 90% confidence level, what is the smallest sample size n that meets the requirements?

A) 411 B) 423 C) 435 D) 447 E) 459
F) 471 G) 483 H) 495 I) 507 J) 519

Solution. *Here* $z_{0.05} = 1.644854$ *and* $2\,ME_0 = 0.08$. *The required sample size is* $n = \lceil (1.644854/0.08)^2 \rceil = \lceil 422.7414 \rceil = 423$ *rounded up to the nest whole number.*

Answer: ***B***

19. In a random sample of 20 outage reports, an electric utility finds that service was restored in less than 2 hours in 16 instances. (Consequently, service failed to be restored in less than 2 hours in only 4 instances.) Find a 95% confidence interval for the proportion of service restoration responses that were completed in less than 2 hours. What is the upper endpoint of the confidence interval?

A) 0.8000 B) 0.8137 C) 0.8274 D) 0.8411 E) 0.8548
F) 0.8684 G) 0.8821 H) 0.8958 I) 0.9095 J) 0.9232

Solution. *The 10-10 Success-Failure Condition is not satisfied. Therefore, the Agresti-Coull adjustment is needed. With the four phony trials included, we have* $n' = 20 + 4 = 24$, $\widehat{p}' = (16+2)/24 = 0.75$, $\widehat{q}' = 1 - \widehat{p}' = (4+2)/24 = 0.25$, $SE(\widehat{p}') = \sqrt{(0.75)(0.25)/24} = 0.08838835$, $ME(\widehat{p}') = z_{0.025} \times SE(\widehat{p}') = (1.959964)(0.08838835) = 0.173238$, *and* $\widehat{p}' \pm ME(\widehat{p}')$ *is the interval* $[0.576762, 0.923238]$.
Answer: ***J***

20. In the manufacture of an engine crankshaft, one sectional diameter is supposed to be 4.00 in, but each production run can "drift" away from the target. This drift does not affect the standard deviation, which is known to be $\sigma = 0.0300$ in. However, because of the drift, the population mean diameter μ for a large production run must be considerd to be unknown. A random sample of size 50 is selected

from the crankshafts produced in the run and their diameters are accurately measured. If the observed sample mean diameter is 3.9812 in, what is the upper endpoint of a 99% confidence interval for μ.

A) 3.8883 B) 3.9229 C) 3.9575 D) 3.9921 E) 4.0267
F) 4.0613 G) 4.0959 H) 4.1305 I) 4.1651 J) 4.1997

Solution. *Because* $n = 50, \widehat{\mu} = 3.9812, \sigma = 0.03,$ *and* $z_{0.005} = 2.575829,$ *the required upper bound is* $\widehat{\mu} + z_{0.005}\,\sigma/\sqrt{n},$ *or* $3.9812 + 2.575829 \times 0.0300/\sqrt{50},$ *or 3.9921.*
Answer: ***D***

21. Let μ denote the mean male Etruscan skull breadth. In a random sample of 84 male Etruscan skulls, the observed sample mean breadth was 143.75 mm with observed sample standard deviation equal to 5.9331 mm. Find the upper endpoint of a 90% confidence interval for μ.

A) 144.0543 B) 144.2064 C) 144.3585 D) 144.5106 E) 144.6627
F) 144.6627 G) 144.8148 H) 144.9669 I) 145.2711 J) 145.4232

Solution. *Because* $n = 84, \widehat{\mu} = 143.75, S = 5.9331,$ *and* $z_{0.05} = 1.644854,$ *the required upper bound is* $\widehat{\mu} + z_{0.05}\,S/\sqrt{n},$ *or* $143.75 + 1.644854 \times 5.9331/\sqrt{84},$ *or 144.8148.*
Answer: ***G***

22. Normal body temperature is, need it be said, normally distributed. In a *small* sample of size 15, the observed sample mean body temperature was 98.349 (degrees Fahrenheit) and the observed sample standard deviation 0.733. What is the upper endpoint of a 95% confidence interval for body temperature?

A) 98.55 B) 98.60 C) 98.65 D) 98.70 E) 98.75
F) 98.80 G) 98.85 H) 98.90 I) 98.95 J) 99.00

Solution. *For this small sample (*$n = 15$*) from a normal distribution, we must use a Student-t distribution with* $n - 1$*, or 14, degrees of freedom. From the Student-t table, we have* $t_{0.025,14} = 2.1448$*. The sample mean is 98.349 and the sample standard deviation is S = 0.733. The standard error is* $S/\sqrt{n}$*, or* $0.733/\sqrt{15}$*, or 0.1892598. The margin of error is* 2.1448×0.1892598*, or 0.4059244. The upper endpoint of a 95% confidence interval is 98.349 + 0.4059244, or 98.75492.*

Answer: ***E***

23. It is desirable that a machine at a bottling plant fills plastic bottles with 16 oz of fluid and with a variance that is as small as possible. To estimate the standard deviation, the fill weights of 20 bottles were measured. The result was a sample standard deviation of 0.3937 oz. Assuming that the fill weight is normally distributed, what is the upper endpoint of a 95% confidence interval for the true variance σ^2 of the fill weight?

A) 0.1726 B) 0.1901 C) 0.2077 D) 0.2253 E) 0.2428
F) 0.2604 G) 0.2780 H) 0.2956 I) 0.3131 J) 0.3307

Solution. *In this problem* $n = 20, S = 0.3937, S^2 = 0.1550, \alpha = 0.05,$ *and* $\chi^2_{1-\alpha/2,n-1} = \chi^2_{0.975,19} = 8.9065$*. A* $100(1-\alpha)\%$ *confidence interval is*

$$\left[\frac{n-1}{\chi^2_{\alpha/2,n-1}}\,S^2, \frac{n-1}{\chi^2_{1-\alpha/2,n-1}}\,S^2\right].$$

The required upper bound is $(n-1)\,S^2/\chi^2_{1-\alpha/2,n-1}$*, or* $19\,(0.1550)/8.9065$*, or 0.3307.*

Answer: ***J***

24. Horseshoe crabs on Delaware Bay beaches are counted every year. The table below shows the numbers in 2011 and 2012 for four randomly selected beaches.

Beach	2011	2012
North Cape May	32610	4350
Villas	56260	32140
Reeds	62179	81503
Kitts Hummock	117360	68400

What is the upper endpoint, rounded up to the nearest greater integer, of a 90% confidence interval for the mean change from 2011 to 2012? (The number we seek is positive if the crab population is, on average, greater in 2012, and negative if the crab population is, on average, greater in 2011. Assume that the number of crabs on a beach can be well-approximated by a normal distribution without the need for a continuity correction. Assume that the numbers found on different beaches are independent. Do *not* assume that the numbers found on the same beach in different years are independent.)

A) -14878 B) -9251 C) -3624 D) 2002 E) 7628
F) 13255 G) 18882 H) 24508 I) 30134 J) 35761

Solution. *Let X and Y denote the number of horseshoe crabs found on a beach in 2011 and 2012 respectively. The differences W = Y - X on the four beaches from 2011 to 2012 are* $Y_1-X_1 = 4350-32610 = -28260$, $Y_2-X_2 = 32140-56260 = -24120$, $Y_3-X_3 = 81503-62179 = 19324$, $Y_4-X_4 = 68400-117360 = -48960$. *We calculate* $\overline{W} = -20504$ *and* $S = Sd(W) = 28689.4$. *The upper endpoint of a 90% confidence interval for the mean change from 2011 to 2012 is*

$$\overline{W} + t_{0.95,3}\,\frac{S}{\sqrt{4}} = -20504 + 2.353363 \times \frac{28689.4}{2} = 13254.29.$$

Answer: ***F***

25. Deinopis and Menneus are species of the spider family Deinopidae. Let X and Y be the respective sizes in mm of their next suppers. An arachnologist specializing in the munching habits of these two species made the following obervations based on samples of size $n = n_X = 10$ and $m = n_Y = 12$: $\overline{X} = 10.24, S_X = 2.47, \overline{Y} = 9.05, S_Y = 1.91$. Find the upper endpoint of a 95% confidence interval for $\mu_X - \mu_Y$. Be conservative. Do not assume that $\sigma_X = \sigma_Y$.

A) 1.3961 B) 1.6407 C) 1.8853 D) 2.1299 E) 2.3745
F) 2.6191 G) 2.8637 H) 3.1083 I) 3.3529 J) 3.5975

Solution. *The required upper endpoint of a 95% confidence interval for* $\mu_X - \mu_Y$ *is*

$$\overline{X} - \overline{Y} + t_{\alpha/2,df} \times \sqrt{\frac{S_X^2}{n} + \frac{S_Y^2}{m}}$$

where df is the minimum of $n-1$ *and* $m-1$. *The required value is* $10.24 - 9.05 + t_{9,0.025}\,\sqrt{(2.47)^2/10 + (1.91)^2/12}$, *or* $1.19 + 2.2622\,\sqrt{0.9140983333}$*, or 3.352855.*

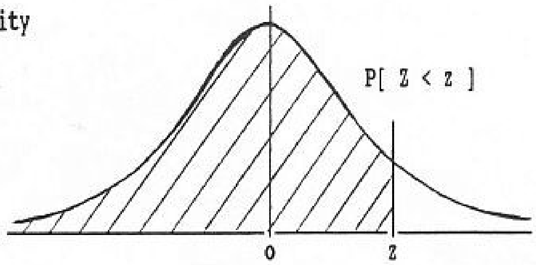
Answer: ***I***

STANDARD STATISTICAL TABLES

1. Areas under the Normal Distribution

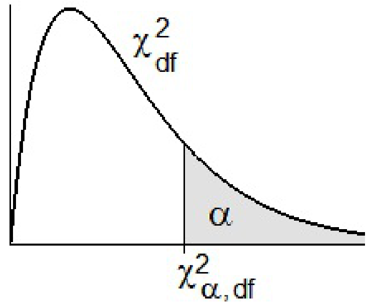
The table gives the cumulative probability up to the standardised normal value z i.e.

$$P[\, Z < z \,] = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp(-\tfrac{1}{2}Z^2)\, dZ$$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7020	0.7054	0.7089	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8804	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9865	0.9868	0.9871	0.9874	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9924	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9980	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986

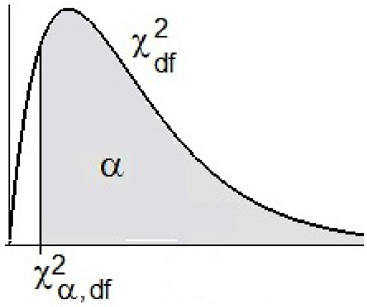
z	3.00	3.10	3.20	3.30	3.40	3.50	3.60	3.70	3.80	3.90
P	0.9986	0.9990	0.9993	0.9995	0.9997	0.9998	0.9998	0.9999	0.9999	1.0000



Values of $\chi^2_{\alpha, df}$ $P(\chi^2_{df} \geq \chi^2_{\alpha, df}) = \alpha$

df \ α	0.005	0.010	0.025	0.050	0.100	0.200	0.250	0.300	0.400	0.500
1	7.8794	6.6349	5.0239	3.8415	2.7055	1.6424	1.3233	1.0742	0.7083	0.4549
2	10.5966	9.2103	7.3778	5.9915	4.6052	3.2189	2.7726	2.4079	1.8326	1.3863
3	12.8382	11.3449	9.3484	7.8147	6.2514	4.6416	4.1083	3.6649	2.9462	2.3660
4	14.8603	13.2767	11.1433	9.4877	7.7794	5.9886	5.3853	4.8784	4.0446	3.3567
5	16.7496	15.0863	12.8325	11.0705	9.2364	7.2893	6.6257	6.0644	5.1319	4.3515
6	18.5476	16.8119	14.4494	12.5916	10.6446	8.5581	7.8408	7.2311	6.2108	5.3481
7	20.2777	18.4753	16.0128	14.0671	12.0170	9.8032	9.0371	8.3834	7.2832	6.3458
8	21.9550	20.0902	17.5345	15.5073	13.3616	11.0301	10.2189	9.5245	8.3505	7.3441
9	23.5894	21.6660	19.0228	16.9190	14.6837	12.2421	11.3888	10.6564	9.4136	8.3428
10	25.1882	23.2093	20.4832	18.3070	15.9872	13.4420	12.5489	11.7807	10.4732	9.3418
11	26.7568	24.7250	21.9200	19.6751	17.2750	14.6314	13.7007	12.8987	11.5298	10.3410
12	28.2995	26.2170	23.3367	21.0261	18.5493	15.8120	14.8454	14.0111	12.5838	11.3403
13	29.8195	27.6882	24.7356	22.3620	19.8119	16.9848	15.9839	15.1187	13.6356	12.3398
14	31.3193	29.1412	26.1189	23.6848	21.0641	18.1508	17.1169	16.2221	14.6853	13.3393
15	32.8013	30.5779	27.4884	24.9958	22.3071	19.3107	18.2451	17.3217	15.7332	14.3389
16	34.2672	31.9999	28.8454	26.2962	23.5418	20.4651	19.3689	18.4179	16.7795	15.3385
17	35.7185	33.4087	30.1910	27.5871	24.7690	21.6146	20.4887	19.5110	17.8244	16.3382
18	37.1565	34.8053	31.5264	28.8693	25.9894	22.7595	21.6049	20.6014	18.8679	17.3379
19	38.5823	36.1909	32.8523	30.1435	27.2036	23.9004	22.7178	21.6891	19.9102	18.3377
20	39.9968	37.5662	34.1696	31.4104	28.4120	25.0375	23.8277	22.7745	20.9514	19.3374
21	41.4011	38.9322	35.4789	32.6706	29.6151	26.1711	24.9348	23.8578	21.9915	20.3372
22	42.7957	40.2894	36.7807	33.9244	30.8133	27.3015	26.0393	24.9390	23.0307	21.3370
23	44.1813	41.6384	38.0756	35.1725	32.0069	28.4288	27.1413	26.0184	24.0689	22.3369
24	45.5585	42.9798	39.3641	36.4150	33.1962	29.5533	28.2412	27.0960	25.1063	23.3367
25	46.9279	44.3141	40.6465	37.6525	34.3816	30.6752	29.3389	28.1719	26.1430	24.3366
30	53.6720	50.8922	46.9792	43.7730	40.2560	36.2502	34.7997	33.5302	31.3159	29.3360
40	66.7660	63.6907	59.3417	55.7585	51.8051	47.2685	45.6160	44.1649	41.6222	39.3353
50	79.4900	76.1539	71.4202	67.5048	63.1671	58.1638	56.3336	54.7228	51.8916	49.3349
60	91.9517	88.3794	83.2977	79.0819	74.3970	68.9721	66.9815	65.2265	62.1348	59.3347
70	104.2149	100.4252	95.0232	90.5312	85.5270	79.7146	77.5767	75.6893	72.3583	69.3345
80	116.3211	112.3288	106.6286	101.8795	96.5782	90.4053	88.1303	86.1197	82.5663	79.3343
90	128.2989	124.1163	118.1359	113.1453	107.5650	101.0537	98.6499	96.5238	92.7614	89.3342
100	140.1695	135.8067	129.5612	124.3421	118.4980	111.6667	109.1412	106.9058	102.9459	99.3341

Chi-Squared Values—Left Tails.



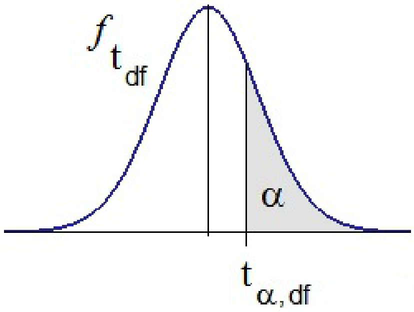
Values of $\chi^2_{\alpha,df}$ $\quad P(\chi^2_{df} \geq \chi^2_{\alpha,df}) = \alpha$

df \ α	0.600	0.700	0.750	0.800	0.900	0.950	0.975	0.990	0.995
1	0.2750	0.1485	0.1015	0.0642	0.0158	0.0039	0.0010	0.0002	0.0000
2	1.0217	0.7133	0.5754	0.4463	0.2107	0.1026	0.0506	0.0201	0.0100
3	1.8692	1.4237	1.2125	1.0052	0.5844	0.3518	0.2158	0.1148	0.0717
4	2.7528	2.1947	1.9226	1.6488	1.0636	0.7107	0.4844	0.2971	0.2070
5	3.6555	2.9999	2.6746	2.3425	1.6103	1.1455	0.8312	0.5543	0.4117
6	4.5702	3.8276	3.4546	3.0701	2.2041	1.6354	1.2373	0.8721	0.6757
7	5.4932	4.6713	4.2549	3.8223	2.8331	2.1673	1.6899	1.2390	0.9893
8	6.4226	5.5274	5.0706	4.5936	3.4895	2.7326	2.1797	1.6465	1.3444
9	7.3570	6.3933	5.8988	5.3801	4.1682	3.3251	2.7004	2.0879	1.7349
10	8.2955	7.2672	6.7372	6.1791	4.8652	3.9403	3.2470	2.5582	2.1559
11	9.2373	8.1479	7.5841	6.9887	5.5778	4.5748	3.8157	3.0535	2.6032
12	10.1820	9.0343	8.4384	7.8073	6.3038	5.2260	4.4038	3.5706	3.0738
13	11.1291	9.9257	9.2991	8.6339	7.0415	5.8919	5.0088	4.1069	3.5650
14	12.0785	10.8215	10.1653	9.4673	7.7895	6.5706	5.6287	4.6604	4.0747
15	13.0297	11.7212	11.0365	10.3070	8.5468	7.2609	6.2621	5.2293	4.6009
16	13.9827	12.6243	11.9122	11.1521	9.3122	7.9616	6.9077	5.8122	5.1422
17	14.9373	13.5307	12.7919	12.0023	10.0852	8.6718	7.5642	6.4078	5.6972
18	15.8932	14.4399	13.6753	12.8570	10.8649	9.3905	8.2307	7.0149	6.2648
19	16.8504	15.3517	14.5620	13.7158	11.6509	10.1170	8.9065	7.6327	6.8440
20	17.8088	16.2659	15.4518	14.5784	12.4426	10.8508	9.5908	8.2604	7.4338
21	18.7683	17.1823	16.3444	15.4446	13.2396	11.5913	10.2829	8.8972	8.0337
22	19.7288	18.1007	17.2396	16.3140	14.0415	12.3380	10.9823	9.5425	8.6427
23	20.6902	19.0211	18.1373	17.1865	14.8480	13.0905	11.6886	10.1957	9.2604
24	21.6525	19.9432	19.0373	18.0618	15.6587	13.8484	12.4012	10.8564	9.8862
25	22.6156	20.8670	19.9393	18.9398	16.4734	14.6114	13.1197	11.5240	10.5197
30	27.4416	25.5078	24.4776	23.3641	20.5992	18.4927	16.7908	14.9535	13.7867
40	37.1340	34.8719	33.6603	32.3450	29.0505	26.5093	24.4330	22.1643	20.7065
50	46.8638	44.3133	42.9421	41.4492	37.6886	34.7643	32.3574	29.7067	27.9907
60	56.6200	53.8091	52.2938	50.6406	46.4589	43.1880	40.4817	37.4849	35.5345
70	66.3961	63.3460	61.6983	59.8978	55.3289	51.7393	48.7576	45.4417	43.2752
80	76.1879	72.9153	71.1445	69.2069	64.2778	60.3915	57.1532	53.5401	51.1719
90	85.9925	82.5111	80.6247	78.5584	73.2911	69.1260	65.6466	61.7541	59.1963
100	95.8078	92.1289	90.1332	87.9453	82.3581	77.9295	74.2219	70.0649	67.3276

Chi-Squared Values—Central Hump + Left Tails.

Values of $t_{\alpha, df}$

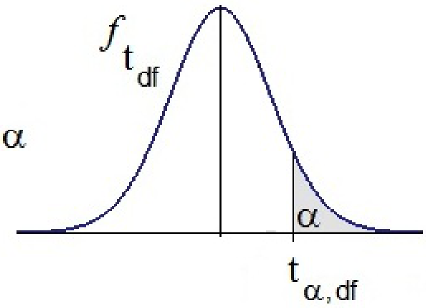
$P(t_{df} \geq t_{\alpha, df}) = \alpha$



df \ α	.450	.400	.350	.300	.250	.200
1	.1584	.3249	.5095	.7265	1.0000	1.3764
2	.1421	.2887	.4447	.6172	.8165	1.0607
3	.1366	.2767	.4242	.5844	.7649	.9785
4	.1338	.2707	.4142	.5686	.7407	.9410
5	.1322	.2672	.4082	.5594	.7267	.9195
6	.1311	.2648	.4043	.5534	.7176	.9057
7	.1303	.2632	.4015	.5491	.7111	.8960
8	.1297	.2619	.3995	.5459	.7064	.8889
9	.1293	.2610	.3979	.5435	.7027	.8834
10	.1289	.2602	.3966	.5415	.6998	.8791
11	.1286	.2596	.3956	.5399	.6974	.8755
12	.1283	.2590	.3947	.5386	.6955	.8726
13	.1281	.2586	.3940	.5375	.6938	.8702
14	.1280	.2582	.3933	.5366	.6924	.8681
15	.1278	.2579	.3928	.5357	.6912	.8662
16	.1277	.2576	.3923	.5350	.6901	.8647
17	.1276	.2573	.3919	.5344	.6892	.8633
18	.1274	.2571	.3915	.5338	.6884	.8620
19	.1274	.2569	.3912	.5333	.6876	.8610
20	.1273	.2567	.3909	.5329	.6870	.8600
21	.1272	.2566	.3906	.5325	.6864	.8591
22	.1271	.2564	.3904	.5321	.6858	.8583
23	.1271	.2563	.3902	.5317	.6853	.8575
24	.1270	.2562	.3900	.5314	.6848	.8569
25	.1269	.2561	.3898	.5312	.6844	.8562
26	.1269	.2560	.3896	.5309	.6840	.8557
27	.1268	.2559	.3894	.5306	.6837	.8551
28	.1268	.2558	.3893	.5304	.6834	.8546
29	.1268	.2557	.3892	.5302	.6830	.8542
30	.1267	.2556	.3890	.5300	.6828	.8538
40	.1265	.2550	.3881	.5286	.6807	.8507
50	.1263	.2547	.3875	.5278	.6794	.8489
60	.1262	.2545	.3872	.5272	.6786	.8477
70	.1261	.2543	.3869	.5268	.6780	.8468
80	.1261	.2542	.3867	.5265	.6776	.8461
90	.1260	.2541	.3866	.5263	.6772	.8456
100	.1260	.2540	.3864	.5261	.6770	.8452

Values of $t_{\alpha, df}$

$P(t_{df} \geq t_{\alpha, df}) = \alpha$



df \ α	.150	.100	.050	.025	.010	.005
1	1.9626	3.0777	6.3138	12.7062	31.8205	63.6567
2	1.3862	1.8856	2.9200	4.3027	6.9646	9.9248
3	1.2498	1.6377	2.3534	3.1824	4.5407	5.8409
4	1.1896	1.5332	2.1318	2.7764	3.7469	4.6041
5	1.1558	1.4759	2.0150	2.5706	3.3649	4.0321
6	1.1342	1.4398	1.9432	2.4469	3.1427	3.7074
7	1.1192	1.4149	1.8946	2.3646	2.9980	3.4995
8	1.1081	1.3968	1.8595	2.3060	2.8965	3.3554
9	1.0997	1.3830	1.8331	2.2622	2.8214	3.2498
10	1.0931	1.3722	1.8125	2.2281	2.7638	3.1693
11	1.0877	1.3634	1.7959	2.2010	2.7181	3.1058
12	1.0832	1.3562	1.7823	2.1788	2.6810	3.0545
13	1.0795	1.3502	1.7709	2.1604	2.6503	3.0123
14	1.0763	1.3450	1.7613	2.1448	2.6245	2.9768
15	1.0735	1.3406	1.7531	2.1314	2.6025	2.9467
16	1.0711	1.3368	1.7459	2.1199	2.5835	2.9208
17	1.0690	1.3334	1.7396	2.1098	2.5669	2.8982
18	1.0672	1.3304	1.7341	2.1009	2.5524	2.8784
19	1.0655	1.3277	1.7291	2.0930	2.5395	2.8609
20	1.0640	1.3253	1.7247	2.0860	2.5280	2.8453
21	1.0627	1.3232	1.7207	2.0796	2.5176	2.8314
22	1.0614	1.3212	1.7171	2.0739	2.5083	2.8188
23	1.0603	1.3195	1.7139	2.0687	2.4999	2.8073
24	1.0593	1.3178	1.7109	2.0639	2.4922	2.7969
25	1.0584	1.3163	1.7081	2.0595	2.4851	2.7874
26	1.0575	1.3150	1.7056	2.0555	2.4786	2.7787
27	1.0567	1.3137	1.7033	2.0518	2.4727	2.7707
28	1.0560	1.3125	1.7011	2.0484	2.4671	2.7633
29	1.0553	1.3114	1.6991	2.0452	2.4620	2.7564
30	1.0547	1.3104	1.6973	2.0423	2.4573	2.7500
40	1.0500	1.3031	1.6839	2.0211	2.4233	2.7045
50	1.0473	1.2987	1.6759	2.0086	2.4033	2.6778
60	1.0455	1.2958	1.6706	2.0003	2.3901	2.6603
70	1.0442	1.2938	1.6669	1.9944	2.3808	2.6479
80	1.0432	1.2922	1.6641	1.9901	2.3739	2.6387
90	1.0424	1.2910	1.6620	1.9867	2.3685	2.6316
100	1.0418	1.2901	1.6602	1.9840	2.3642	2.6259