

# Homework 6

Math 217

Due: 24 October 2018 by 11:59 PM

**Instructions:** Write your solutions to the following problems and submit them on Crowdmark by the deadline. You are encouraged to work in groups or consult with each other on the problems, but the work submitted must be your own and must be written up by you.

(1) Compute the convolution of  $f(t) = e^{at}$  and  $g(t) = e^{bt}$  (with  $a \neq b$ ) in two different ways: first by the definition, and secondly by using the Laplace transform.

(2) Find the inverse Laplace transform of

$$F(s) = \ln \left( \frac{s^2 + 9}{s^2 - 4} \right).$$

(3) Find any nonzero solution to

$$tx'' + (t - 4)x' + x = 0$$

such that  $x(0) = 0$ .

(4) Consider the differential equation

$$x'' + 2x' + x = f(t)$$

with conditions  $x(0) = x'(0) = 0$ . Show that the solution is given by

$$x(t) = \int_0^t \tau e^{-\tau} f(t - \tau) d\tau.$$