

Please present your solutions clearly and in an organized way. Answer the questions in the space provided on the question sheets. If you run out of room for an answer, continue on the back of the page. **Please note that the use of a calculator is not allowed.** Good luck!! 😊

Full Name: _____

Question	Points	Score
1	20	
2	20	
3	35	
4	20	
5	25	
6	20	
Total:	140	

This exam has 6 questions, for a total of 140 points. The maximum possible score for each problem is given on the right side of the problem.

This is blank space. If you are bored, you can draw something (e.g., your favorite Pokémon).



1. Calculate the following. You do not need to show your calculations.

(a) $\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

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(b) $\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 2 & 0 \end{bmatrix}$

5

(c) $\begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

5

(d) $\begin{bmatrix} 3 \\ 4 \end{bmatrix} \begin{bmatrix} 2 & 1 \end{bmatrix}$

5

2. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix}$.

(a) Is A invertible? If so, what is A^{-1} ? (Hint: You should be able to fit the calculations in the space provided.)

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(b) What is $\text{im}(A)$?

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(c) What is $\text{ker}(A)$?

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This is blank space. If you are bored, you can draw something (e.g., your favorite superhero).



3. Let $A = \begin{bmatrix} 1 & 3 & 2 \\ 3 & 9 & 6 \end{bmatrix}$.

(a) What is $\text{im}(A)$? There are many acceptable ways to answer this (e.g., with a picture or with a geometric description).

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(b) Find $\text{rref}(A)$.

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(c) What are all the solutions to the following system?

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$$\begin{aligned}x + 3y + 2z &= 3 \\ 3x + 9y + 6z &= -1\end{aligned}$$

(d) What are all the solutions to the following system?

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$$\begin{aligned}x + 3y + 2z &= 0 \\ 3x + 9y + 6z &= 0\end{aligned}$$

This is blank space. If you are bored, you can draw something (e.g., your favorite character from a cartoon/game/movie/book/etc.).



4. Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$, $T(\vec{x}) = A\vec{x}$, where $A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$.

(a) Give a geometric description of this transformation.

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(b) Give a geometric description of the inverse T^{-1} .

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(c) What is the matrix A^{-1} ?

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5. Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the linear transformation which rotates points **clockwise** by 90 degrees around the origin.

(a) What is $T\left(\begin{bmatrix} 1 \\ 4 \end{bmatrix}\right)$? (Maybe drawing a picture would help.)

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(b) What is the matrix A for which $T(\vec{x}) = A\vec{x}$?

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(c) What is A^{40} ?

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This is blank space. If you are bored, you can draw something (e.g., your favorite mathematical figure/object/function).



6. Determine if the following functions are linear transformations or not.

(a) $T : \mathbb{R}^1 \rightarrow \mathbb{R}^1, T(x) = 4x - 2$

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(b) $T : \mathbb{R}^2 \rightarrow \mathbb{R}^4, T(\vec{x}) = \vec{0}$. (In other words, $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$.)

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This is blank space. If you are bored, you can draw something. I am out of ideas.