Math 19620/20 lectures outline

I will update this document after every lecture to keep track of what we covered, and to indicate what I plan to cover in the next lecture.

Week 1

10/2/18. Sections 1.1, 1.2

- Gauss-Jordan elimination
- reduced row-echelon form

10/4/18. Sections 1.2, 1.3

- geometric interpretation of linear system, as intersection of $(m-1)$-planes in $\mathbb{R}^n$
- rank of a matrix (relationship to number of free variables)
- Theorem 1.3.4 in textbook
- basic vector operations
- geometric interpretation of linear system, involving vector operations

Week 2

10/9/18. Section 1.3, 2.1

- vector form of linear system, why we may have no solutions or infinitely many solutions
- multiplication of matrix and vector

10/11/18. Section 2.1

- linear transformations
- Theorems 1.3.10, 2.1.2, and Theorem 2.1.3
- some examples
  - $T(x) = x^2$ is not linear
  - $T(\begin{bmatrix} x \\ y \end{bmatrix}) = 5x - 3y$ is linear
  - $T(\begin{bmatrix} x \\ y \end{bmatrix}) = \begin{bmatrix} x+1 \\ y \end{bmatrix}$ is not linear
  - $T(\vec{x}) = \ell\vec{x}$ is linear. If $T : \mathbb{R}^2 \to \mathbb{R}^2$, the matrix is $\begin{pmatrix} \ell & 0 \\ 0 & \ell \end{pmatrix}$.

Week 3

10/16/18. Section 2.2, 2.3

- geometric transformation: scaling, rotation, reflection, projection, shearing
- how to find the matrices for these transformations. (Don't worry about the matrices for reflection and for projection.)
- composition of linear transformations

10/18/18. Section 2.3

- sum of linear transformations, and matrix addition
- composition of linear transformations, and matrix multiplication
- identity function, and identity matrix

Week 4

10/23/18. Section 2.4

- inverse functions and inverse matrices
- how to find inverse matrices (Theorem 2.4.5)
- inverse of 2×2 matrices

10/25/18. Section 3.1

- image of a linear transformation
- definition of span
- properties of the image (Theorem 3.1.4)
- kernel of a linear transformation

Week 5

10/30/18 (planned). Section 3.1, Section 3.2

- More on the kernel of a linear transformation
- Subspaces of $\mathbb{R}^n$, redundancy, linear independence

11/1/18.

- midterm...

Week 6

11/6/18. Section 3.3

- Span, linear independence, basis
- Dimension of a subspace
- Using rref to find basis for kernel and image

11/8/18. Section 3.3 (finish up), Section 3.4 (very brief)

- Rank-nullity theorem,
- Coordinates, how to go from the standard coordinate system to a different coordinate system
- linear transformations with respect to different coordinates
- Example in $\mathbb{R}^2$: Orthogonal projection onto any line

Week 7

11/13/18. Section 5.1

- Review of dot products
- Orthogonal vectors, orthonormal basis
- Projections when we have an orthonormal basis
- The matrix of an orthogonal projection (Theorem 5.3.10)

11/15/18. Section 5.2, 5.4

- Gram-Schmidt process (skip QR factorization)
- Least-squares solution, normal equation

Week 8

11/20/18. Section 5.4, Chapter 6 (very briefly)

- applications of least-squares solutions to fitting data
- formula for 2x2 and 3x3 determinants
- know this fact: A square matrix A is invertible if and only if $\det A \neq 0$.
- some "applications" of determinants (not useful in practice)
- geometric interpretation of determinants

Week 9

11/27/18. Section 7.1

- dynamical system with coyotes and roadrunners (Example 7), phase portraits
- definition of eigenvectors, eigenvalues, eigenbasis (Definition 7.1.2)
- eigenbases and diagonlization (Theorem 7.1.3)

11/29/18. Section 7.1, 7.2, 7.3 (only the important parts)

- eigenvectors and eigenvalues of some geometric transformations
- how to find eigenvectors and eigenvalues and eigenbasis (Look at Theorem 7.3.7 for a summary.)

Week 10

12/4/18. Section 7.4

- Applications to Markov chains,
- Distribution vectors, transition matrices, equilibrium distribution
- Regular transition matrices have equilibrium distributions (Theorem 7.4.1)
- Example with 3 cities (Section 7.4, Example 1)
- Application to PageRank

That's all!