MATH 15300/14

Midterm #2

FEBRUARY 20, 2020
ALAN CHANG

Please present your solutions clearly and in an organized way. Simplify all your final answers. If an answer box is given, write your final answer in the box. If you run out of room, continue on the extra pages provided at the end. **The use of a calculator is not allowed.** Good luck!! ⌣̈

Full Name:

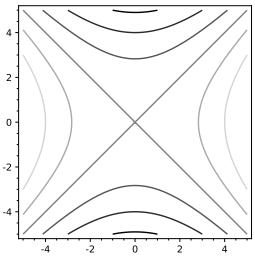
Student ID:

Question	Points	Score
1	9	
2	10	
3	20	
4	20	
5	22	
6	10	
7	10	
Total:	101	

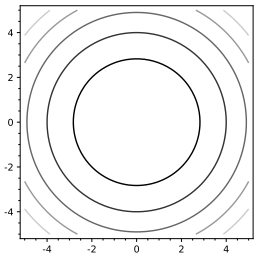
This exam has 7 questions, for a total of 101 points. The maximum possible score for each problem is given on the right side of the problem.

Please do not open this exam packet unless you are instructed to do so.

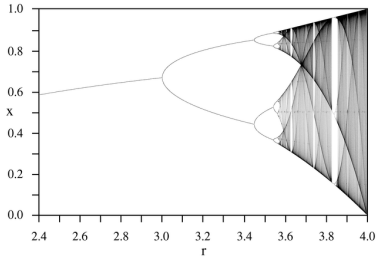
1. Match the plots to the descriptions below by writing the appropriate letter (A, B, C) in the boxes. (No justification needed.) [9]



(A)



(B)



(C)

1. The bifurcation diagram for the logistic map ☐
2. The level curves of the function $f(x,y) = x^2 + y^2$ ☐
3. The level curves of the function $f(x,y) = x^2 - y^2$ ☐

2. Find an equation for the plane which passes through (3,2,0) and is perpendicular to the vector (2,-1,1). Write your final answer in the form $Ax + By + Cz = D$, where A, B, C, D are numbers. [10]

equation of plane: ☐

3. Determine whether the series converges or diverges. Please justify.

(a) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^{1/10}}$ circle one: CONVERGES DIVERGES

10

(b) $\displaystyle\sum_{k=1}^{\infty} \frac{3k+1}{\sqrt{k^4+k+1}}$ circle one: CONVERGES DIVERGES

10

4. If possible, give an example of a series satisfying the given properties. If it is impossible, no explanation is needed.

(a) $\sum a_k$ converges and $\sum |a_k|$ converges. circle one: POSSIBLE IMPOSSIBLE 5

(b) $\sum a_k$ converges and $\sum |a_k|$ diverges. circle one: POSSIBLE IMPOSSIBLE 5

(c) $\sum a_k$ diverges and $\sum |a_k|$ converges. circle one: POSSIBLE IMPOSSIBLE 5

(d) $\sum a_k$ diverges and $\sum |a_k|$ diverges. circle one: POSSIBLE IMPOSSIBLE 5

5. (a) For the following four statements, circle either TRUE or FALSE. No explanation is needed. (Hint: Exactly two are TRUE.) [12]

If $a_k \to 0$, then $\sum a_k$ converges.	TRUE	FALSE
If $\sum a_k$ diverges, then $a_k \not\to 0$.	TRUE	FALSE
If $a_k \not\to 0$, then $\sum a_k$ diverges.	TRUE	FALSE
If $\sum a_k$ converges, then $a_k \to 0$.	TRUE	FALSE

(b) Show that if $\sum a_k$ converges, then $\sum \dfrac{(a_k+1)^{15300}}{a_k^2+2020}$ diverges. (Hint: Use part (a).) [10]

10

6. Determine whether the lines ℓ_1 and ℓ_2 are parallel, skew, or intersecting. If they intersect, find the point of intersection.

ℓ_1 : $x_1(t) = 2 + t$, $y_1(t) = -1 - t$, $z_1(t) = 3 + 2t$

ℓ_2 : $x_2(u) = 1 + u$, $y_2(u) = 1 - 2u$, $z_2(u) = 1 + 2u$

10

7. Suppose $\vec{a}$ and $\vec{b}$ are two perpendicular vectors. Using vector operations, prove the following:

$$\|\vec{a} + \vec{b}\|^2 = \|\vec{a}\|^2 + \|\vec{b}\|^2$$

(Hint: Start by expanding the left-hand side.)

(Fun fact: This is a way of proving the Pythagorean theorem.)

This is blank space. If you are bored, try to draw a Pringle®.