MATH 15200/14

Midterm #2

NOVEMBER 12, 2019
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Please present your solutions clearly and in an organized way. Simplify all your final answers. If an answer box is given, write your final answer in the box. If you run out of room, continue on the extra pages provided at the end. **The use of a calculator is not allowed.** Good luck!! ☺

Full Name: sample solutions

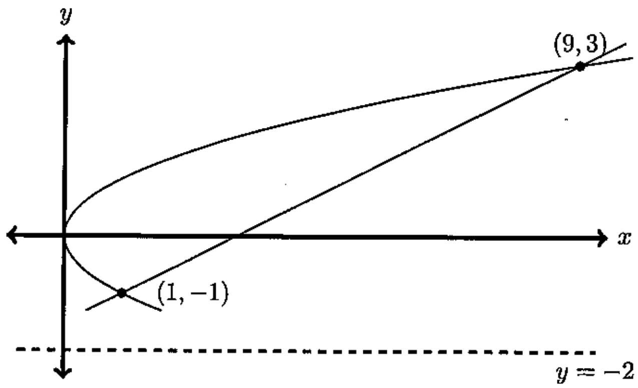
Student ID:

Question	Points	Score
1	20	
2	30	
3	10	
4	20	
5	20	
6	20	
Total:	120	

This exam has 6 questions, for a total of 120 points. The maximum possible score for each problem is given on the right side of the problem.

Please do not open this exam packet unless you are instructed to do so.

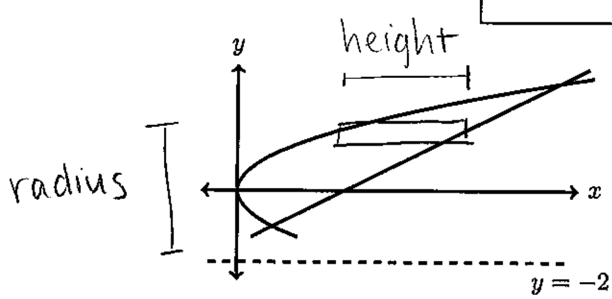
1.



The region bounded by $x = y^2$ and $x = 2y + 3$ is revolved about the line $y = -2$.

(a) Give an expression for the volume of this solid using integrals obtained by the **shell method.** You do not have to simplify the integrands or evaluate the integrals. In the diagram provided below, draw a thin rectangle that is used in the calculations. $\boxed{10}$

integral expression = $\displaystyle\int_{-1}^{3} 2\pi(y+2)(2y+3-y^2)\,dy$

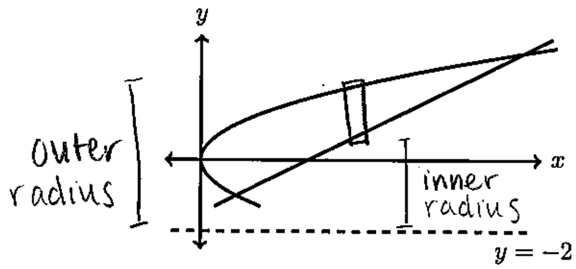


radius = $y+2$

height = $2y+3-y^2$

(b) Give an expression for the volume of this solid using integrals obtained by the **disk/washer method.** You do not have to simplify the integrands or evaluate the integrals. In the diagram provided below, draw a thin rectangle that is used in the calculations. $\boxed{10}$

integral expression = $\boxed{\int_0^1 \pi\left[(\sqrt{x}+2)^2-(-\sqrt{x}+2)^2\right]dx + \int_1^9 \pi\left[(\sqrt{x}+2)^2-\left(\frac{x-3}{2}+2\right)^2\right]dx}$



$x = y^2$

$\pm\sqrt{x} = y$

$x = 2y+3$

$x-3 = 2y$

$\frac{x-3}{2} = y$

For $0 \le x \le 1$: outer radius $= \sqrt{x}+2$

inner radius $= -\sqrt{x}+2$

For $1 \le x \le 9$: outer radius $= \sqrt{x}+2$

inner radius $= \frac{x-3}{2}+2$

2. Evaluate the following.

(a) $\dfrac{d}{dx}\displaystyle\int_0^{x^3} (1+t^2)^4\,dt = \boxed{3x^2(1+x^6)^4}$ 10

$$= (1+(x^3)^2)^4 \frac{d}{dx}[x^3]$$

$$= (1+x^6)^4(3x^2)$$

$$= 3x^2(1+x^6)^4$$

(b) $\displaystyle\int_1^e \frac{(\ln x)^2}{x}\,dx = \boxed{1/3}$ 10

$u = \ln x$ upper $= \ln e = 1$

$du = \frac{1}{x}\,dx$ lower $= \ln 1 = 0$

$$= \int_0^1 u^2\,du = \left[\frac{u^3}{3}\right]_0^1 = \frac{1}{3}$$

(c) $\displaystyle\int \frac{6y^2+2}{y^3+y+4}\,dy = \boxed{2\ln|y^3+y+4| + C}$ 10

$u = y^3+y+4$

$du = (3y^2+1)\,dy$

$= \displaystyle\int \frac{2}{u}\,du$

$= 2\ln|u| + C$

$= 2\ln|y^3+y+4| + C$

3. Without using the fundamental theorem of calculus, show that $\displaystyle\int_{200}^{500} \frac{dx}{x} = \int_2^5 \frac{dx}{x}$. 10

Start with $\displaystyle\int_{200}^{500} \frac{dx}{x}$.

Let $u = \frac{1}{100}x$ upper $= \frac{500}{100} = 5$

$du = \frac{1}{100}\,dx$ lower $= \frac{200}{100} = 2$.

$$\int_{200}^{500} \frac{dx}{x} = \int_2^5 \frac{100\,du}{100\,u} = \int_2^5 \frac{du}{u} = \int_2^5 \frac{dx}{x}$$

4. Let $f(x) = 4 - x - x^{25}$.

(a) Show that f is one-to-one. 10

$$f'(x) = -1 - 25x^{24} \leq -1$$

f' is strictly negative

$\Rightarrow$ f is decreasing

$\Rightarrow$ f is one-to-one

(b) Evaluate: $(f^{-1})'(2) = \boxed{-1/26}$ 10

$$(f^{-1})'(2) = \frac{1}{f'(f^{-1}(2))}$$

$f^{-1}(2) = ?$

$4 - x - x^{25} = 2$

$\Rightarrow x = 1$

so $f^{-1}(2) = 1$

$$= \frac{1}{f'(1)}$$

$$= \frac{1}{-1-25(1)^{24}}$$

$$= -\frac{1}{26}$$

5. (a) Find the average value of $f(x) = 3x^2$ on the interval $[-1, 2]$. 10

average value = $\boxed{3}$

$$\frac{1}{2-(-1)} \int_{-1}^{2} 3x^2 \, dx = \frac{1}{3} \left[x^3 \right]_{-1}^{2}$$

$$= \frac{1}{3} \left[2^3 - (-1)^3 \right] = \frac{1}{3} [8 + 1]$$

$$= \frac{1}{3} [9] = 3$$

(b) Find the average value of $f(x) = \dfrac{x^7}{1+x^2}$ on the interval $[-2, 2]$. 10

average value = $\boxed{0}$

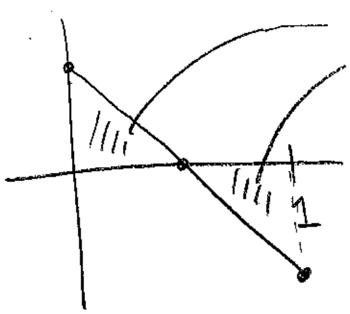
$$\frac{1}{2-(-2)} \int_{-2}^{2} \frac{x^7}{1+x^2} \, dx = 0$$

since $\dfrac{x^7}{1+x^2}$ is an odd function

6. Sketch the graph of a function f that is continuous on $[0,1]$ and meets the given conditions. If this is not possible, give a reason why.

(a) $\int_0^1 f(x)\,dx = 0$ and $\int_0^1 |f(x)|\,dx \neq 0$. $\boxed{10}$

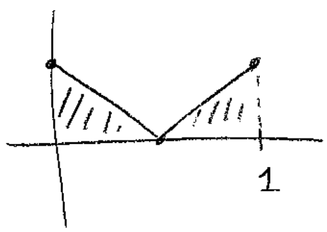
f:



these two regions have the same area,

so $\int_0^1 f(x)\,dx = 0$

|f|:



the shaded regions are above the x-axis,

so $\int_0^1 |f(x)|\,dx > 0$

(b) $\int_0^1 f(x)\,dx \neq 0$ and $\int_0^1 |f(x)|\,dx = 0$. $\boxed{10}$

Not possible. If $\int_0^1 |f(x)|\,dx = 0$, then f must be the constant function $f(x) = 0$,

so $\int_0^1 f(x)\,dx$ must also be 0.

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Name a Pokémon: Sonic the Hedgehog?