

Please present your solutions clearly and in an organized way. Simplify all your final answers. If an answer box is given, write your final answer in the box. If you run out of room, continue on the extra pages provided at the end. **The use of a calculator is not allowed.** Good luck!! 😊

Full Name:

Student ID:

Question	Points	Score
1	40	
2	24	
3	15	
4	15	
5	20	
6	10	
7	20	
Total:	144	

This exam has 7 questions, for a total of 144 points. The maximum possible score for each problem is given on the right side of the problem.



1. Evaluate the following expressions. Show your work.

(a)  $\int \left( \frac{4x^3 + 1}{x^2} + 3 \cos x \right) dx =$

10

(b)  $\int_0^4 \frac{4x}{\sqrt{x^2 + 9}} dx =$

10



$$(c) \frac{d}{dt} \int_t^1 (y^2 + 1)^3 dy =$$

10

$$(d) \int_0^1 \frac{d}{dt} [(t^2 + 1)^3] dt =$$

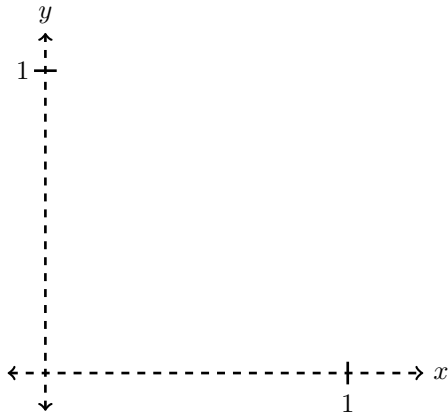
10



2. Sketch the graph of a function  $f$  that is **defined on**  $[0, 1]$  and meets the given conditions. If this is not possible, give a reason why.

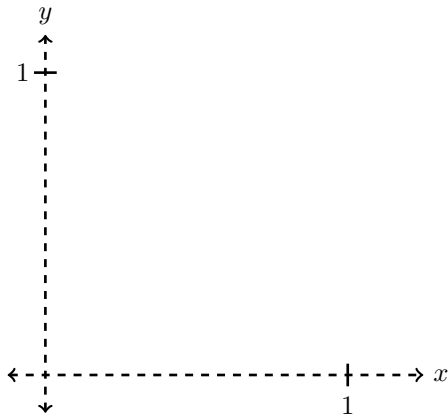
- (a)  $f$  is continuous on  $(0, 1)$   
 $f$  takes on the values 0 and 1  
 $f$  does **NOT** take on the value  $\frac{1}{2}$

6



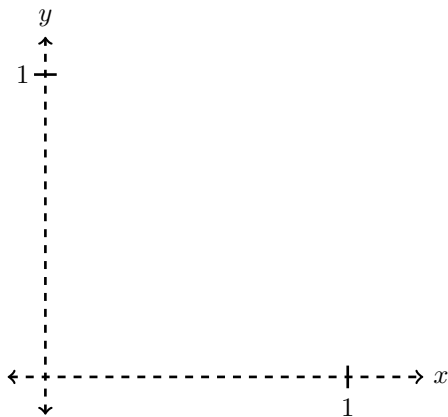
- (b)  $f$  is continuous on  $(0, 1]$   
 $f$  does **NOT** take on a maximum value

6



- (c)  $f$  is continuous on  $[0, 1]$   
 $f$  does **NOT** take on a maximum value

6



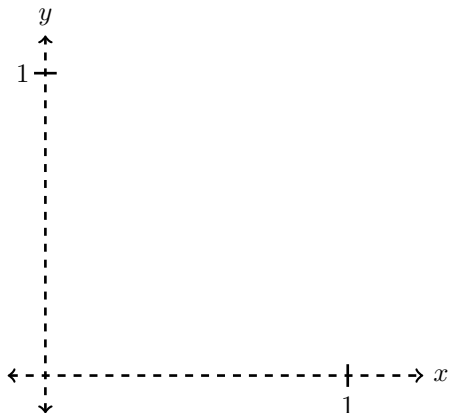


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(d)  $f$  is continuous on  $[0, 1]$

$$f(0) = f(1)$$

There does **NOT** exist a point  $c \in (0, 1)$  such that  $f'(c) = 0$



3. Show that  $\lim_{x \rightarrow 2} (5 - \frac{1}{2}x) = 4$  using the  $(\epsilon, \delta)$  definition of the limit.

15



4. Let  $f(x) = x^2 + 1$ . Consider the partition  $P = \{-2, 1, 3\}$ .

- (a) Calculate the lower Riemann sum  $L_f(P)$ . Include a sketch of the function and the relevant rectangles. 10

$$L_f(P) =$$

- (b) Write  $<$  (less than) or  $=$  (equals to) or  $>$  (greater than) into the box below to indicate the relation between the two quantities. You do not need to justify. 5

$$L_f(P) \quad \square \quad \int_{-2}^3 (x^2 + 1) dx$$



5. Consider the following statement:

"If  $0 < |x - 3| < \delta$ , then  $|x^2 - 9| < 7$ ."

(a) Find a  $\delta > 0$  so that the statement above is **TRUE**. Please justify with a sketch and a brief explanation.

10

$\delta =$

(b) Find a  $\delta > 0$  so that the statement above is **FALSE**. Please justify with a sketch and a brief explanation.

10

$\delta =$



6. Consider the functions  $f(x) = x^2 - 2$  and  $g(x) = -2x^2 + 1$ .

Sketch the region bounded by the two curves, and find the area of that region.

area =





7. Consider the integral  $\int_0^1 \sqrt{1-x^2} dx$ .

In this problem, we will evaluate this integral in two different ways.

- (a) Give a geometric interpretation of the integral as the area of some region in the  $xy$ -plane. Please include a sketch. Use basic (i.e., high school level) geometry to evaluate the integral.

10

$$\int_0^1 \sqrt{1-x^2} dx = \boxed{\phantom{0000000000}}$$

(b) Fact:  $\frac{d}{dx} [x\sqrt{1-x^2} + \arcsin x] = 2\sqrt{1-x^2}$ .

10

Use this fact (which you do not need to prove) and the fundamental theorem of calculus to evaluate the integral.

$$\int_0^1 \sqrt{1-x^2} dx = \boxed{\phantom{0000000000}}$$



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This is even more blank space.

If you are bored, you can draw something (e.g. your favorite Pokémon).