Math 11200/20 worksheet
Friday, September 30, 2016

Problem 1

Consider the following question: "Let $\star$ be the operation defined by $a \star b = (2ab - a - b)^2$. Find $1 \star (2 \star 3)$."

(a) What is the answer?

(b) Is $\star$ commutative? Is it associative?

(c) Is $\star$ a function? If so, what are its domain and range?

Problem 2

Let's work with "one's digit arithmetic." Consider the set $\mathbb{Z}_{10} = \{0, 1, \ldots, 9\}$. We can add and multiply, e.g.,

$$1 + 2 = 3, \qquad 6 + 9 = 5, \qquad 8 + 2 = 0, \qquad 2 \cdot 3 = 6, \qquad 5 \cdot 7 = 5, \qquad 2 \cdot 0 = 0$$

Which of the properties A1–A4, M1–M4, D hold?

Problem 3

Which elements of $\mathbb{Z}_{10}$ have additive inverses? Which elements of $\mathbb{Z}_{10}$ have multiplicative inverses?

Problem 4

Can you make sense of any of these in $\mathbb{Z}_{10}$?

(a) $5 - 2, \qquad 3 - 5, \qquad 0 - 4, \qquad 1/2, \qquad 1/3, \qquad 1/4, \qquad 2/2, \qquad 3/2, \qquad 4/2$

(b) $\sqrt{1}, \qquad \sqrt{2}, \qquad \sqrt{3}, \qquad \sqrt{4}, \qquad \sqrt{5}, \qquad \sqrt{6}, \qquad \sqrt[3]{1}, \qquad \sqrt[3]{2}, \qquad \sqrt[3]{3}, \qquad \sqrt[3]{4}$

(c) $\log_3 7, \qquad \log_5 2, \qquad \log_2 5$

(d) Can you solve $x^2 + x + 4 = 0$ in $\mathbb{Z}_{10}$? Does the quadratic formula work?