Math 11200/20 homework 6
Due date: Friday, November 11, 2016

Note: You shouldn't need to use a calculator for these problems.

Please present your solutions clearly and in an organized way. Think of it this way: if you show it to another student in this class, he/she should be able to understand it without needing to ask you questions.

**Problem 6.1.** Exercise 4.4 from the textbook. (You can leave your answer as a prime factorization if you'd like.)

**Problem 6.2.** For each part of Exercise 4.4 of the textbook, find the LCMs. (You can leave your answer as a prime factorization if you'd like.)

**Problem 6.3.** Find the number of divisors of the following numbers:

(a) 51
(b) 234
(c) 272
(d) 180
(e) 9261

Hint: You don't actually have to write out all the divisors. Use the prime factorization to help you!

**Problem 6.4.** What can you deduce about the prime factorization of $n$ if:

(a) $n$ has exactly 2 divisors? (Here, we're only referring to positive divisors.)
(b) $n$ has exactly 3 divisors?
(c) $n$ has exactly 4 divisors?
(d) $n$ has exactly 5 divisors?

**Problem 6.5.** Let $a, b \in \mathbb{Z}$. Show that if $a^2 \mid b^2$, then $a \mid b$.
Hint: use the prime factorizations! Let $a = p_1^{a_1} \cdots p_k^{a_k}$ and $b = p_1^{b_1} \cdots p_k^{b_k}$. (Note that it's okay for exponents to be zero.) If $a^2 \mid b^2$, then what can we say about the exponents?

**Problem 6.6.** Let $F_1 = 1$, $F_2 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 3$. (These are the Fibonacci numbers.) Show that

$$F_2 + F_4 + F_6 + \cdots + F_{2n} = F_{2n+1} - 1$$

(Hint: Recall in class that $1 + 2 + 4 + \cdots + 2^n = 2^{n+1} - 1$.)

**Problem 6.7.** Can you find three integers $a, x, y \in \mathbb{Z}$ such that $x \equiv y \pmod{10}$ but $a^x \not\equiv a^y \pmod{10}$? (You can use a calculator.)

**Problem 6.8.** What are all the solutions to $3x \equiv 4 \pmod{10}$? If do some trial and error, we see that the answer is $x \in \{\ldots, -12, -2, 8, 18, 28, \ldots\}$. In other words, the answer is $x \equiv 8 \pmod{10}$.
Do some trial and error to find all solutions to the following. You do not need to justify.

(a) What are all $x \in \mathbb{Z}$ which satisfy $4x \equiv 6 \pmod{10}$?
(b) What are all $x \in \mathbb{Z}$ which satisfy BOTH $x \equiv 0 \pmod{4}$ and $x \equiv 3 \pmod{5}$?