Math 11200/20 homework 3
Due date: Friday, October 14, 2016

Note: You shouldn't need to use a calculator for these problems.

Please present your solutions clearly and in an organized way. Think of it this way: if you show it to another student in this class, he/she should be able to understand it without needing to ask you questions.

Problem 3.1. Suppose S is a set with a binary operation $+$ satisfying axioms A1–A4. Using the axioms, show the following:

$$\text{For all } a, b \in S,\ -(a+b) = (-a) + (-b).$$

Note that this equation $-(a+b) = (-a)+(-b)$ means: "the additive inverse of $a+b$ is equal to the sum of the additive inverse of a and the additive inverse of b."
(Hint: Take a look at the proofs of theorem 2.3 and 2.4 in the textbook.)

Problem 3.2. Exercise 3.7 in the textbook. It looks like a lot of problems, but they're all very short. For example, solve (a) by yourself and then see the footnote for a sample write-up.[1]

Problem 3.3. Exercise 3.8 in the textbook.

Problem 3.4. Prove that in $\mathbb{Z}$, if $a \mid b$ and $b > 0$, then $a \leq b$. Turn the scratch-work from class into an actual proof. (Near or at the beginning, the proof should say something like "We break into two cases: (1) if $a \leq 0$, and (2) if $a > 0$.")

Problem 3.5.

(a) In $\mathbb{Z}_{10}$, what are the elements with multiplicative inverses? What about in $\mathbb{Z}_{12}$? In $\mathbb{Z}_{16}$? $\mathbb{Z}_2$? $\mathbb{Z}_3$? $\mathbb{Z}_4$? $\mathbb{Z}_5$? (You don't need to show your calculations; you can just list out the elements.)

(b) In these examples, if a has a multiplicative inverse in $\mathbb{Z}_n$, what do you notice about the GCD of a and n? What if a does not have a multiplicative inverse? (You can just state your observations; you don't need to prove anything.)

Problem 3.6. Exercise 3.9 in the textbook. (Note: the notation (a, b) does not mean an ordered pair, but instead the GCD of a and b.)

Problem 3.7. Exercise 3.10 in the textbook.

[1] The following would be enough:

"True. Suppose $a \mid b$. Then there's a $k \in \mathbb{Z}$ such that $ak = b$. Then $4ak = 4b$, so $(2a)(2k) = 4b$, so $2a \mid 4b$."