Derivatives on the TI-83

We have already seen situations where $f'(a)$, the derivative of a function $f(x)$ at the number a, must be estimated— for example,

    1) when we don't have a formula for $f(x)$ (for example, see Ex. 4, pp. 147-148)
    2) when we have a formula for $f(x)$ but still can't figure out the value of $\lim\limits_{h \to 0} \frac{f(a+h)-f(a)}{h}$

    to get the exact answer   (for example, see Ex. 3, pp. 152-153).

Most calculators, including the TI-83, are not clever enough to find the exact value for $f'(a)$ and always give you a numeric approximation to the exact answer instead. Generally, these approximations are quite accurate, and the error is so small that it causes no difficulties in practical applications. However, if you're don't understand what the calculator is doing when it computes a derivative, it can mislead you (sometimes badly!!). The discussion below is about the TI-83, but the general principles and warnings apply to most calculators.

The TI-83's command for differentiation is *nDeriv*, on the "Math" menu. For example, the command $nDeriv(\sqrt{x}, x, 4)$ finds $f'(4)$, where $f(x) = \sqrt{x}$. The TI-83 reports the answer .250000002.
However, we know the exact answer: from Example 4, p. 161, $f'(x) = \frac{1}{2\sqrt{x}}$, so $f'(4)$ is exactly $\frac{1}{4}$!

The TI-83's answer is slightly off. (*Most other calculators will produce similar sorts of errors, but other calculators may not show exactly this same error).* The calculator's answer is only a numeric approximation, rather than an answer obtained by substituting into a formula for $f'(x)$ (the "n" in *nDeriv* stands for "numeric").

How does the TI-83 make its estimate? Because $f'(a) = \lim\limits_{h \to 0} \frac{f(a+h)-f(a)}{h}$, substituting an h value near 0 into $\frac{f(a+h)-f(a)}{h}$ should give a reasonable estimate for $f'(a)$. The TI-83
(automatically) chooses two such h values, $h = .001$ and $h = -.001$, makes two estimates in that way, and averages them to get its answer. Thus, for $f(x)=\sqrt{x}$, the TI-83 estimated
$$f'(4) \simeq \frac{1}{2}\left[\frac{f(4+.001)-f(4)}{.001} + \frac{f(4-.001)-f(4)}{-.001}\right], \text{ and got } .250000002 .$$

This method is often quite accurate, but can lead to terrible errors, even in very simple situations, if you don't use the calculator intelligently. For example, suppose $f(x) = \frac{1}{x}$. This function has no derivative at 0 — it isn't even defined there!! (Using the limit to find the derivative $f'(x)$, we find
that $f'(x) = -\frac{1}{x^2}$, which we also see doesn't make sense when $x = 0$.) But if you ask the TI-83 to do the calculation to find the derivative at 0— using the command *nDeriv(1/x, x, 0)*— it blindly uses the method above and returns the answer 1000000 !!! (*Use the method above and explain how the TI-83 got its answer*). The point is that *you* should know (from calculus) that this function has no derivative at 0 and not ask for it !

Notes: 1) When you use the nDeriv command, you can force the TI-83 to use an h value different from $h = \pm .001$ by adding that into the differentiation command. To use $h = \pm .00001$, for example, use the command nDeriv($\sqrt{x}$, x, 4, .00001)
        2) If you have entered $f(x)$ as Y1, you can define $f'(x)$ as Y2 = nDeriv(Y1,X,X), $f''(x)$ as Y3 = nDeriv(Y2,X,X), etc. Then the TI-83 can be used to graph these functions as well.