Name Peter Townsend

Math 132 Quiz
8 AM - 9 AM

1. Is the series
$$\sum_{n=1}^{\infty} \frac{2^{2n}+3^n}{n2^{2n}}$$
convergent or is it divergent? Justify your answer

$$a_n = \frac{2^{2n}+3^n}{n2^{2n}} = \frac{4^n+3^n}{n4^n}, \text{ limit compare w/ } b_n = \frac{4^n}{n4^n} = \frac{1}{n}$$

$$L = \lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{4^n+3^n}{n4^n}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{4^n+3^n}{4^n} = \lim_{n\to\infty} 1+\left(\frac{3}{4}\right)^n = 1$$

$0 < L < \infty \Rightarrow$ both $\sum a_n$ and $\sum b_n$ converge or diverge

Since $\sum b_n = \sum \frac{1}{n}$ diverges, $\sum a_n$ diverges

2. Let
$$S = \sum_{n=1}^{\infty} (-1)^n \frac{1}{n^3} \quad \text{and} \quad S_N = \sum_{n=1}^{N} (-1)^n \frac{1}{n^3}.$$
For what N can you be sure that $|S - S_N| < 0.001$?

Consider $|S - S_N| \le |a_{N+1}| < .001$

$$\Rightarrow \quad |a_{N+1}| = \left|(-1)^{N+1}\frac{1}{(N+1)^3}\right| = \frac{1}{(N+1)^3} < \frac{1}{1000}$$

$$\Rightarrow \quad 1000 < (N+1)^3$$

If $N=9$, $(9+1)^3 = 1000 \not> 1000$

If $N=10$, $(10+1)^3 = 1331 > 1000$

So $N \geq 10$ works

Name Peter Townsend

Math 132 Quiz
9 AM - 10 AM

1. Is the series

$$\sum_{n=1}^{\infty} \frac{3^{2n} + 5^n}{\sqrt{n}3^{2n}}$$

convergent or is it divergent? Justify your answer

$a_n = \frac{3^{2n}+5^n}{\sqrt{n}\,3^{2n}} = \frac{9^n+5^n}{\sqrt{n}\,9^n}$, limit compare w/ $b_n = \frac{9^n}{\sqrt{n}\,9^n} = \frac{1}{\sqrt{n}}$

$L = \lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{9^n+5^n}{\sqrt{n}\,9^n}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty} \frac{9^n+5^n}{9^n} = \lim_{n\to\infty} 1+\left(\frac{5}{9}\right)^n = 1$

$0 < L < \infty \Rightarrow$ Both Σa_n and Σb_n converge or diverge

$\Sigma b_n = \Sigma \frac{1}{\sqrt{n}}$ diverges by p-series

So Σa_n diverge

2. Let

$$S = \sum_{n=1}^{\infty} (-1)^n \frac{1}{n^2+20} \qquad \text{and} \qquad S_N = \sum_{n=1}^{N} (-1)^n \frac{1}{n^2+20}.$$

For what N can you be sure that $|S - S_N| < 0.01$?

Consider $|S - S_N| \le |a_{N+1}| < .01$

$\Rightarrow |a_{N+1}| = \left|(-1)^{N+1} \cdot \frac{1}{(N+1)^2+20}\right| = \frac{1}{(N+1)^2+20} < \frac{1}{100}$

$\Rightarrow 100 < (N+1)^2 + 20$

$\Rightarrow 80 < (N+1)^2$

If $N=6$, $(6+1)^2 = 49 \not> 80$ So $N \ge 8$ works

If $N=7$, $(7+1)^2 = 64 \not> 80$

If $N=8$, $(8+1)^2 = 81 > 80$

Name Peter Townsend

Math 132 Quiz
12 Noon - 1 PM

1. Is the series
$$\sum_{n=1}^{\infty} \frac{n2^n + \ln(n)}{n3^n + 1}$$
convergent or is it divergent?. Justify your answer

Let $a_n = \frac{n2^n + \ln n}{n3^n + 1}$, $b_n = \frac{n2^n}{n3^n} = \frac{2^n}{3^n}$

Then $L = \lim_{n\to\infty} \frac{\frac{n2^n + \ln n}{n3^n+1}}{\frac{2^n}{3^n}} = \lim_{n\to\infty} \frac{n + \frac{\ln n}{2^n}}{n + \frac{1}{3^n}} = \lim_{n\to\infty} \frac{n}{n} = 1$

$\sum b_n$ conv. by geo. series, so $\sum a_n$ conv. by limit comparison

Note: Cannot use comparison test with this b_n: $\sum b_n$ converges, but is $a_n < b_n$ for almost all n?

$a_n < b_n \Rightarrow \frac{n2^n + \ln n}{n3^n+1} < \frac{2^n}{3^n} \iff n6^n + 3^n \ln n < n6^n + 2^n$

$\iff 3^n \ln n < 2^n$, which is not true.

However, for $n \geq 1$,

$\frac{n2^n + \ln n}{n3^n + 1} < \frac{n2^n + \ln n}{n3^n}$

$< \frac{n2^n + n}{n3^n}$

$= \left(\frac{2}{3}\right)^n + \left(\frac{1}{3}\right)^n$

whose sum converges

2. Let
$$S = \sum_{n=1}^{\infty} (-1)^n \frac{1}{2n^2+1} \quad \text{and} \quad S_N = \sum_{n=1}^{N} (-1)^n \frac{1}{2n^2+1}.$$
For what N can you be sure that $|S - S_N| < 0.01$?

Consider $|S - S_N| \leq |a_{N+1}| < .01$:

$|a_{N+1}| = \frac{1}{2(N+1)^2+1} < \frac{1}{100}$

$\Rightarrow 2(N+1)^2 + 1 > 100$

Try: $N=6$: $2(6+1)^2+1 = 99 \not> 100$

$N=7$: $2(7+1)^2+1 = 129 > 100$

So $N \geq 7$ works