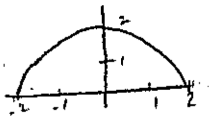
Peter
Townsend

Math 132 Quiz
8 AM - 9 AM

1. Find the y-coordinate of the center of mass of the semicircular region $0 \leq y \leq \sqrt{4-x^2}$. (You do not need to show any work for the calculation of the area of the region.)

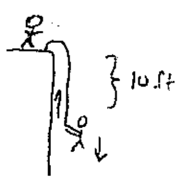


$$\bar{y} = \frac{\frac{1}{2}\int_{-2}^{2}\left(\sqrt{4-x^2}\right)^2 dx}{\frac{1}{2}\pi(2)^2} = \frac{\int_{-2}^{2}(4-x^2)\,dx}{4\pi}$$

$$= \frac{1}{4\pi}\left[4x - \frac{1}{3}x^3\right]_{-2}^{2} = \frac{1}{4\pi}\left[\left(8-\frac{8}{3}\right)-\left(-8+\frac{8}{3}\right)\right]$$

$$= \frac{32/3}{4\pi} = \frac{8}{3\pi}$$

2. A 150 pound roofer bumps into a barrel of tar, tips it over, and falls off the roof. He is able to grasp the end of a rope that is secured to the roof-top and that dangles 10 feet down the side of the building. Because of the spilled tar trickling down, the weight density of the rope and the tarry mess adhering to it is $2+y$ pounds per foot where y is the distance in feet from the top of the roof $(0 \leq y \leq 10)$. Muttering to himself "I do all the work around here," a second roofer pulls up the rope and his clumsy partner. How much work did he do?



$$W_{roofer} = 10 \cdot 150 = 1500$$

$$W_{rope} = \int_0^{10} (2+y)\,y\,dy$$

$$= \int_0^{10} (2y+y^2)\,dy$$

$$= y^2 + \frac{1}{3}y^3\Big]_0^{10} = \frac{1000}{3} + 100 = \frac{1300}{3}$$

$$W = 1500 + \frac{1300}{3} = \frac{5800}{3}$$

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Math 132 Quiz
9 AM - 10 AM

1. Find the y-coordinate of the center of mass of the semicircular region $0 \le y \le \sqrt{1-x^2}$. (You do not need to show any work for the calculation of the area of the region.)

$$\bar{y} = \frac{\frac{1}{2}\int_{-1}^{1} \left(\sqrt{1-x^2}\right)^2 dx}{\frac{1}{2}\cdot\pi(1)^2} = \frac{\int_{-1}^{1}(1-x^2)\,dx}{\pi} = \frac{1}{\pi}\left[x - \frac{1}{3}x^3\right]_{-1}^{1}$$

$$= \frac{1}{\pi}\left[\left(1-\frac{1}{3}\right) - \left(-1+\frac{1}{3}\right)\right] = \frac{1}{\pi}\left[\frac{4}{3}\right] = \frac{4}{3\pi}$$

2. A 200 pound roofer bumps into a barrel of tar, tips it over, and falls off the roof. He is able to grasp the end of a rope that is secured to the roof-top and that dangles 10 feet down the side of the building. Because of the spilled tar trickling down, the weight density of the rope and the tarry mess adhering to it is $1 + 2y$ pounds per foot where y is the distance in feet from the top of the roof ($0 \le y \le 10$). Muttering to himself "I do all the work around here," a second roofer pulls up the rope and his clumsy partner. How much work did he do?

$$W_{roofer} = 10 \cdot 200 = 2000$$

$$W_{rope} = \int_0^{10}(1+2y)\,y\,dy = \int_0^{10}(y+2y^2)\,dy$$

$$= \frac{1}{2}y^2 + \frac{2}{3}y^3\Big]_0^{10} = 50 + \frac{2000}{3} = \frac{2150}{3}$$

$$W = 2000 + \frac{2150}{3} = \frac{8150}{3}$$

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Math 132 Quiz
12 Noon - 1 PM

1. The area of the region $0 \le y \le \sqrt{x - x^2}$, $0 \le x \le 1$ is $\pi/8$. Find the y-coordinate of the center of mass of this region.

$$\bar{y} = \frac{\frac{1}{2}\int_0^1 \left(\sqrt{x-x^2}\right)^2 dx}{\pi/8} = \frac{1}{2}\cdot\frac{8}{\pi}\int_0^1 (x-x^2)\,dx$$

$$= \frac{4}{\pi}\left(\frac{1}{2}x^2 - \frac{1}{3}x^3\right)\Big|_0^1 = \frac{4}{\pi}\left(\frac{1}{2}-\frac{1}{3}\right) = \frac{4}{\pi}\left(\frac{1}{6}\right) = \frac{2}{3\pi}$$

2. A 120 pound roofer bumps into a barrel of tar, tips it over, and falls off the roof. He is able to grasp the end of a rope that is secured to the roof-top and that dangles 20 feet down the side of the building. Because of the spilled tar trickling down, the weight density of the rope and the tarry mess adhering to it is $2 + y/20$ pounds per foot where y is the distance in feet from the top of the roof $(0 \le y \le 20)$. Muttering "I do all the work around here," a second roofer pulls up the rope and his clumsy partner. How much work did he do?

$$W_{roofer} = 120 \cdot 20 = 2400$$

$$W_{rope} = \int_0^{20} \left(2 + \frac{y}{20}\right) y\, dy$$

$$= \int_0^{20} \left(2y + \frac{y^2}{20}\right) dy = y^2 + \frac{y^3}{60}\Big]_0^{20} = 400 + \frac{8000}{60} = \frac{1600}{3}$$

$$W = 2400 + \frac{1600}{3} = \frac{7600}{3}$$