Peter
Townsend

Math 132 Quiz
8 AM - 9 AM

1. Find the arc length of the graph of $y = x^{3/2} - \sqrt{x}/3$ between the points $(1, 2/3)$ and $(4, 22/3)$.

want: $\int_1^4 \left(1 + [y'(x)]^2\right)^{1/2} dx = \int_1^4 \left(1 + \left[\frac{3}{2}x^{1/2} - \frac{1}{6}x^{-1/2}\right]^2\right)^{1/2} dx$

$= \int_1^4 \left(1 + \left[\left(\frac{3}{2}x^{1/2}\right)^2 - \frac{1}{2} + \left(\frac{1}{6}x^{-1/2}\right)^2\right]\right)^{1/2} dx$

$= \int_1^4 \left[\left(\frac{3}{2}x^{1/2}\right)^2 + \frac{1}{2} + \left(\frac{1}{6}x^{-1/2}\right)^2\right]^{1/2} dx$

$= \int_1^4 \left[\left(\frac{3}{2}x^{1/2} + \frac{1}{6}x^{-1/2}\right)^2\right]^{1/2} dx$

$= \int_1^4 \left(\frac{3}{2}x^{1/2} + \frac{1}{6}x^{-1/2}\right) dx$

$= x^{3/2} + \frac{1}{3}x^{1/2} \Big]_1^4 = \left(4^{3/2} + \frac{1}{3}4^{1/2}\right) - \left(1 + \frac{1}{3}\right) = \left(8 + \frac{2}{3}\right) - \left(1 + \frac{1}{3}\right) = 7 + 1/3$

2. If $f(x) = 2/x^2$, $1 \le x \le 2$ is the probability density function of a random variable X, what is the mean μ_X?

want: $\int_1^2 x\, f(x)\, dx = \int_1^2 x \cdot \frac{2}{x^2}\, dx = \int_1^2 \frac{2}{x}\, dx$

$= 2 \ln|x| \Big]_1^2$

$= 2(\ln 2 - \ln 1) = 2 \ln 2$

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Math 132 Quiz
9 AM - 10 AM

1. Find the arc length of the graph of $y = x^2 - \ln(x)/8, \qquad 1 \le x \le 2.$

$$\int_1^2 \left(1+(y'(x))^2\right)^{1/2} dx = \int_1^2 \left(1 + \left(2x - \frac{1}{8x}\right)^2\right)^{1/2} dx$$

$$= \int_1^2 \left[1 + \left((2x)^2 - \frac{1}{2} + \left(\frac{1}{8x}\right)^2\right)\right]^{1/2} dx$$

$$= \int_1^2 \left[(2x)^2 + 1/2 + \left(\frac{1}{8x}\right)^2\right]^{1/2} dx = \int_1^2 \left[\left(2x + \frac{1}{8x}\right)^2\right]^{1/2} dx$$

$$= \int_1^2 \left(2x + \frac{1}{8x}\right) dx = x^2 + \frac{\ln|x|}{8}\Big]_1^2 = \left(4 + \frac{\ln 2}{8}\right) - \left(1 + \frac{\ln 1}{8}\right)$$

$$= 3 + \frac{\ln 2}{8}$$

2. If $f(x) = 4x^3/65, 2 \le x \le 3$ is the probability density function of a random variable X, what is the mean μ_X?

$$\int_2^3 x f(x)\, dx = \int_2^3 \frac{4}{65} x^4\, dx = \frac{4}{65} \cdot \frac{1}{5} x^5 \Big]_2^3$$

$$= \frac{4 \cdot 3^5}{65 \cdot 5} - \frac{4 \cdot 2^5}{65 \cdot 5} = \frac{844}{325}$$

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Math 132 Quiz
NOON - 1 PM

1. Find the arc length of the graph of $y = \ln(x) - x^2/8$, $1 \le x \le 2$.

$$\int_1^2 \left(1 + (y'(x))^2\right)^{1/2} dx = \int_1^2 \left(1 + \left(\frac{1}{x} - \frac{x}{4}\right)^2\right)^{1/2} dx$$

$$= \int_1^2 \left[1 + \left(\left(\frac{1}{x}\right)^2 - \frac{1}{2} + \left(\frac{x}{4}\right)^2\right)\right]^{1/2} dx$$

$$= \int_1^2 \left[\left(\frac{1}{x}\right)^2 + \frac{1}{2} + \left(\frac{x}{4}\right)^2\right]^{1/2} dx = \int_1^2 \left[\left(\frac{1}{x} + \frac{x}{4}\right)^2\right]^{1/2} dx$$

$$= \int_1^2 \left(\frac{1}{x} + \frac{x}{4}\right) dx = \ln x + \frac{x^2}{8} \Big]_1^2 = \ln 2 + \frac{1}{2} - \ln 1 - \frac{1}{8}$$

$$= \ln 2 + \frac{3}{8}$$

2. If $f(x) = 3x^2/7, 1 \le x \le 2$ is the probability density function of a random variable X, what is the mean μ_X?

$$\int_1^2 x f(x)\, dx = \frac{3}{7}\int_1^2 x^3 dx = \frac{3}{28} x^4 \Big]_1^2$$

$$= \frac{3}{28}(16 - 1) = \frac{45}{28}$$