Peter Townsend

Math 132 Quiz
8 AM - 9 AM

1. Calculate

$$\int \frac{3x-12}{x^2-x-2}\,dx.$$

$x^2-x-2 = (x-2)(x+1)$

$$\frac{3x-12}{(x-2)(x+1)} = \frac{A}{x-2} + \frac{B}{x+1} \Rightarrow 3x-12 = A(x+1) + B(x-2)$$

$x=2 \Rightarrow -6 = 3A \Rightarrow A=-2$

$x=-1 \Rightarrow -15 = -3B \Rightarrow B=5$

$$\Rightarrow \int \left(\frac{-2}{x-2} + \frac{5}{x+1}\right) dx = -2\ln|x-2| + 5\ln|x+1| + C$$

2. *Using a trigonometric substitution*, calculate

$$\int \frac{x}{(4-x^2)^{3/2}}\,dx.$$

let $x = 2\sin\theta$

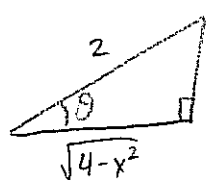
$dx = 2\cos\theta\, d\theta$

$$\Rightarrow \int \frac{2\sin\theta \cdot 2\cos\theta}{(4-4\sin^2\theta)^{3/2}}\, d\theta = \frac{4}{4^{3/2}} \int \frac{\sin\theta\cos\theta}{(\cos^2\theta)^{3/2}}\, d\theta$$

$$= \frac{1}{2}\int \frac{\sin\theta}{\cos^2\theta}\, d\theta \qquad u = \cos\theta,\ du = -\sin\theta\, d\theta$$

$$= \frac{1}{2}\int \frac{-1}{u^2}\, du = \frac{-1}{2}\cdot\frac{-1}{u} + C = \frac{1}{2u} + C = \frac{1}{2\cos\theta} + C$$

$x = 2\sin\theta \Rightarrow \theta = \sin^{-1}\left(\frac{x}{2}\right) \Rightarrow$



$\Rightarrow \cos\theta = \frac{\sqrt{4-x^2}}{2}$

$$\Rightarrow \frac{1}{2\cos\theta} + C = \frac{1}{2\left(\frac{\sqrt{4-x^2}}{2}\right)} + C = \frac{1}{\sqrt{4-x^2}} + C$$

Peter
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Math 132 Quiz
9 AM - 10 AM

1. Calculate

$$\int \frac{7x+11}{x^2+x-2}\,dx.$$

$x^2+x-2 = (x+2)(x-1)$

$$\frac{7x+11}{(x+2)(x-1)} = \frac{A}{x+2} + \frac{B}{x-1} \Rightarrow 7x+11 = A(x-1) + B(x+2)$$

$x=1 \Rightarrow 18 = 3B \Rightarrow B=6$

$x=-2 \Rightarrow -3 = -3A \Rightarrow A=1$

$$\int \left(\frac{1}{x+2} + \frac{6}{x-1}\right)dx = \ln|x+2| + 6\ln|x-1| + C$$

2. *Using a trigonometric substitution*, calculate

$$\int \frac{x}{(25-x^2)^{3/2}}\,dx.$$

$x = 5\sin\theta$

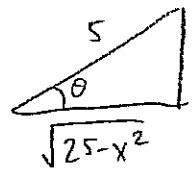
$dx = 5\cos\theta\,d\theta$

$$\Rightarrow \int \frac{5\sin\theta \cdot 5\cos\theta}{(125-25\sin^2\theta)^{3/2}}\,d\theta = \frac{25}{25^{3/2}}\int \frac{\sin\theta\cos\theta}{(\cos^2\theta)^{3/2}}\,d\theta$$

$$= \frac{1}{5}\int \frac{\sin\theta}{\cos^2\theta}\,d\theta, \quad u = \cos\theta, \; du = \sin\theta\,d\theta$$

$$= \frac{1}{5}\int \frac{-1}{u^2}\,d\theta = \frac{-1}{5}\cdot\frac{-1}{u} + C = \frac{1}{5}\cdot\frac{1}{\cos\theta} + C$$

$x = 5\sin\theta \Rightarrow \theta = \sin^{-1}\left(\frac{x}{5}\right) \Rightarrow$



$\Rightarrow \cos\theta = \frac{\sqrt{25-x^2}}{5}$

$$\Rightarrow \frac{1}{5}\cdot\frac{1}{\cos\theta} + C = \frac{1}{5\left(\frac{\sqrt{25-x^2}}{5}\right)} + C = \frac{1}{\sqrt{25-x^2}} + C$$

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Math 132 Quiz
Noon - 1 PM

1. Calculate

$$\int \frac{x-15}{x^2-2x-3}\,dx.$$

$x^2-2x-3 = (x-3)(x+1)$

$$\frac{x-15}{(x-3)(x+1)} = \frac{A}{x-3} + \frac{B}{x+1} \Rightarrow x-15 = A(x+1) + B(x-3)$$

$x=-1 \Rightarrow -16 = -4B \Rightarrow B=4$

$x=3 \Rightarrow -12 = 4A \Rightarrow A=-3$

$$\Rightarrow \int \left(\frac{-3}{x-3} + \frac{4}{x+1}\right) dx = -3\ln|x-3| + 4\ln|x+1| + C$$

2. *Using a trigonometric substitution*, calculate

$$\int \frac{x}{\sqrt{x^2-49}}\,dx.$$

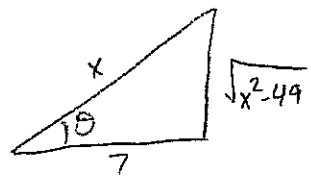
$x = 7\sec\theta$

$dx = 7\sec\theta\tan\theta$

$$\Rightarrow \int \frac{7\sec\theta \cdot 7\sec\theta\tan\theta}{\sqrt{49\sec^2\theta - 49}}\,d\theta = \frac{49}{\sqrt{49}}\int \frac{\sec^2\theta\tan\theta}{\sqrt{\tan^2\theta}}\,d\theta = 7\int \sec^2\theta\,d\theta$$

$$= 7\tan\theta + C$$

$x = 7\sec\theta \Rightarrow \theta = \sec^{-1}\left(\frac{x}{7}\right) \Rightarrow$



$\Rightarrow \tan\theta = \frac{\sqrt{x^2-49}}{7}$

$$\Rightarrow 7\tan\theta + C = 7\cdot\frac{\sqrt{x^2-49}}{7} + C = \sqrt{x^2-49} + C$$