Peter Rodnsen

Math 132 Quiz
8 AM - 9 AM

1. Calculate $\frac{d}{dx} \ln (x)^x$.

$$\ln(x)^x = e^{x \ln(\ln x)}$$

$$\frac{d}{dx}\left(e^{x \ln(\ln x)}\right) = e^{x \ln(\ln x)} \cdot \left(\ln(\ln x) + x \cdot \frac{1}{x \ln x}\right)$$

$$= \ln(x)^x \left(\ln(\ln x) + \frac{1}{\ln x}\right)$$

2. If the instantaneous rate of change of a population at time t (in years) is $\ln(2)/6$ times the value of the population at time t, in how many years does the population increase by a factor of 8?

$k = \frac{\ln 2}{6}$, $k\tau = \ln 2 \Rightarrow \tau = 6$ constant doubling time of $y(t) = Ae^{\frac{\ln 2}{6}t}$, $A = y(0)$

Double three times : $3\tau = 18$

$$= \frac{6 \ln 8}{\ln 2}$$

Peter Townsend

Math 132 Quiz
9 AM - 10 AM

1. Calculate $\frac{d}{dx}\sin(x)^x$.

$$(\sin x)^x = e^{x \ln(\sin x)}$$

$$\frac{d}{dx} e^{x \ln(\sin x)} = e^{x \ln(\sin x)} \left(\ln(\sin x) + \frac{x \cos x}{\sin x} \right)$$

$$= (\sin x)^x \left(\ln(\sin x) + x \cot x \right)$$

2. If the instantaneous rate of change of a population at time t (in years) is $\ln(2)/5$ times the value of the population at time t, in how many years does the population increase by a factor of 16?

$k\tau = \ln 2$, $k = \frac{\ln 2}{5} \Rightarrow \tau = 5$, constant doubling time of $y(t) = Ae^{\frac{\ln 2}{5}t}$, $A = y(0)$

Double four times: $4\tau = 20$

Peter Townsend

Math 132 Quiz
12 Noon - 1 P.M.

1. Calculate $\frac{d}{dx}x^{\tan(x)}$.

$$x^{\tan x} = e^{\tan x(\ln x)}$$

$$\frac{d}{dx}\left(e^{\tan x(\ln x)}\right) = e^{\tan x(\ln x)}\left(\sec^2 x \cdot \ln x + \frac{\tan x}{x}\right)$$

$$= x^{\tan x}\left(\sec^2 x \ln x + \frac{\tan x}{x}\right)$$

2. If, at time t (in years), the instantaneous rate of change of the mass of a radioactive substance is $-\ln(2)/5$ times the value of the mass at time t, in how many years is the mass reduced to one-eighth of the initial amount?

$kT = -\ln 2$, $k = \frac{-\ln 2}{5} \Rightarrow T = 5$, halving time for

$$y(t) = Ae^{\frac{-\ln 2}{5}t}, \quad A = y(0)$$

Halve three times: $3T = 15$