Keto
Tunse)

Math 132 Quiz
8 AM - 9 AM

1. Calculate $\int_0^{\pi/4} \frac{\cos(x)-\sin(x)}{\cos(x)+\sin(x)}\, dx$.

$u = \cos x + \sin x$
$du = \cos x - \sin x \; dx$

$\Rightarrow \int_{x=0}^{x=\pi/4} \frac{1}{u}\, du = \ln(\cos x + \sin x)\Big]_0^{\pi/4}$

$= \ln\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) - \ln(1+0)$

$= \ln(\sqrt{2}) = \frac{\ln(2)}{2}$

2. Use Simpson's Rule with 4 subintervals to approximate the area between the graphs of $y = x^2 - 4x$ and $y = -2x + 3$.

$N = 4$, $f = x^2 - 4x$, $g = -2x + 3$

$f - g = x^2 - 2x - 3 = (x-3)(x+1)$

$f-g$: $- \; - \; - \; -$ on $-1, 0, 1, 2, 3$

$(f-g)(0) = (-3)(1) = -3 < 0$

$\Rightarrow \text{Area} = \int_{-1}^{3} (g-f)(x)\, dx \approx \frac{\Delta x}{3}\left(h(x_0) + 4h(x_1) + 2h(x_2) + 4h(x_3) + h(x_4)\right)$

$\Delta x = 1$, $= \frac{1}{3}\left(h(-1) + 4h(0) + 2h(1) + 4h(2) + h(3)\right)$

$h = g - f = -(x-3)(x+1)$

$= \frac{1}{3}\left((-1)(-4)(0) + (-4)(-3)(1) + (-2)(-2)(2) + (-4)(-1)(3) + (-1)(0)(4)\right)$

$= \frac{1}{3}(0 + 12 + 8 + 12 + 0)$

$= \frac{32}{3}$

Actual: $\int_{-1}^{3} (3 + 2x - x^2)\, dx = 3x + x^2 - \frac{1}{3}x^3 \Big]_{-1}^{3}$

$= (9 + 9 - 9) - (-3 + 1 + \frac{1}{3})$

$= 9 + 5/3 = 10\,2/3 = 32/3$

Peter Townsend

Math 132 Quiz
9 AM - 10 AM

1. Calculate $\int_0^1 \frac{\exp(x)-\exp(-x)}{\exp(x)+\exp(-x)}\,dx$.

$u = e^x + e^{-x}$
$du = e^x - e^{-x}\,dx$

$\int_{x=0}^{x=1} \frac{du}{u} = \ln(e^x + e^{-x})\Big]_0^1$

$= \ln(e + e^{-1}) - \ln(1+1)$

$= \ln(e+e^{-1}) - \ln 2$

$= \ln\left(\frac{e+e^{-1}}{2}\right)$

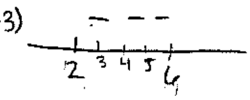
$= \ln(\sinh(1))$

2. Use Simpson's Rule with 4 subintervals to approximate the area between the graphs of $y = 3x^2 - 12x$ and $y = 12x - 36$.

$N=4$, $f = 3x^2 - 12x$ $\quad f-g = 3x^2 - 24x + 36$ $\quad (f-g)(3) = 3(1)(-3)$

$g = 12x - 36$ $\quad = 3(x^2 - 8x + 12)$

$= 3(x-2)(x-6)$



$\Delta x = 1$, $h = g - f = -3(x-2)(x-6)$

want $\int_2^6 (g-f)(x)\,dx \approx \frac{1}{3}\left(h(2) + 4h(3) + 2h(4) + 4h(5) + h(6)\right)$

$= \frac{1}{3}\left((-3)(0)(-4) + (4)(-3)(1)(-3) + (2)(-3)(2)(-2)\right.$

$\left. + (4)(-3)(3)(-1) + (-3)(4)(0)\right)$

$= \frac{1}{3}(0 + 36 + 24 + 36 + 0)$

$= \frac{1}{3}(96) = 32$

Actual: $\int_2^6 (-3x^2 + 24x - 36)\,dx = -x^3 + 12x^2 - 36x\Big]_2^6$

$= (-216 + 432 - 216) - (-8 + 48 - 72)$

$= 0 - (-32) = 32$

Peter Jansen

Math 132 Quiz
12 Noon - 1 P.M.

1. Calculate $\int_0^{\pi/4} \frac{1+\tan^2(x)}{1+\tan(x)}\,dx$.

$\sin^2 x + \cos^2 x = 1$

$\tan^2 x + 1 = \sec^2 x$

$$\int_0^{\pi/4} \frac{\sec^2 x}{1+\tan x}\,dx \qquad u = 1+\tan x, \quad du = \sec^2 x\,dx$$

$$= \int_{x=0}^{x=\pi/4} \frac{1}{u}\,du = \ln(1+\tan x)\Big]_0^{\pi/4}$$

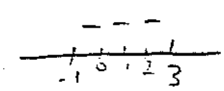
$$= \ln(1+1) - \ln(1)$$

$$= \ln 2$$

2. Use Simpson's Rule with 4 subintervals to approximate the area between the graphs of $y = x^2 - 4x + 1$ and $y = 7 - x^2$.

$N=4, \quad f = x^2-4x+1, \quad g = 7-x^2$

$f-g$:



$\Delta x = 1$

$$f-g = 2x^2-4x-6 = 2(x^2-2x-3)$$

$$= 2(x-3)(x+1)$$

$(f-g)(0) = 2(-3)(1) < 0$, let $h = g-f = -2(x-3)(x+1)$

Want $\int_{-1}^{3} (g-f)(x)\,dx \approx \frac{1}{3}\left(h(-1) + 4h(0) + 2h(1) + 4h(2) + h(3) \right)$

$$= \frac{1}{3}\big((-2)(-4)(0) + (4)(-2)(-3)(1) + (2)(-2)(-2)(2)$$
$$+ (4)(-2)(-1)(3) + (-2)(0)(4) \big)$$

$$= \frac{1}{3}(0 + 24 + 16 + 24 + 0)$$

$$= \frac{1}{3}(64) = \frac{64}{3}$$

Actual: $\int_{-1}^{3} (6+4x-2x^2)\,dx = 6x + 2x^2 - \frac{2}{3}x^3 \Big]_{-1}^{3}$

$$= (18+18-18) - \left(-6+2+\frac{2}{3}\right)$$

$$= \frac{64}{3}$$