Peter Townsend

Math 132 Quiz
8 AM - 9 AM

NAME:________________

1. Is the series
$$\sum_{n=2}^{\infty}(-1)^n\frac{2^n}{n^{1/3}\cdot 3^n}$$
absolutely convergent, is it conditionally convergent, or is it divergent?

Let $L = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\frac{\frac{2^{n+1}}{(n+1)^{1/3}\cdot 3^{n+1}}}{\frac{2^n}{n^{1/3}3^n}} = \lim_{n\to\infty}\frac{2}{3}\cdot\frac{1}{\left(\frac{n+1}{n}\right)^{1/3}} = \frac{2}{3} < 1$

So converges absolutely by ratio test

2. What is the interval of convergence of
$$\sum_{n=1}^{\infty}\frac{2^n(-3-x)^n}{\sqrt{n}}?$$

$\sum_1^{\infty}\frac{2^n(-3-x)^n}{\sqrt{n}} = \sum_1^{\infty}\frac{(-1)^n 2^n(x+3)^n}{\sqrt{n}}$

Let $a_n = \frac{(-1)^n 2^n}{\sqrt{n}}$, then $R = \lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right| = \lim_{n\to\infty}\frac{\frac{2^n}{\sqrt{n}}}{\frac{2^{n+1}}{\sqrt{n+1}}} = \lim_{n\to\infty}\frac{1}{2}\cdot\frac{1}{\sqrt{\frac{n}{n+1}}} = \frac{1}{2}$

So series conv. for $|x+3| \le 1/2$, or $(-3-\frac{1}{2}, -3+1/2) = (-\frac{7}{2}, -\frac{5}{2})$

Check Endpoints:

$x = -7/2$: $\sum_1^{\infty}\frac{(-1)^n 2^n(-7/2+3)^n}{\sqrt{n}} = \sum_1^{\infty}\frac{(-1)^n 2^n(-1/2)^n}{\sqrt{n}} = \sum_1^{\infty}\frac{1}{\sqrt{n}}$, diverges by p-series

$x = -5/2$: $\sum_1^{\infty}\frac{(-1)^n 2^n(-5/2+3)^n}{\sqrt{n}} = \sum_1^{\infty}\frac{(-1)^n 2^n(1/2)^n}{\sqrt{n}} = \sum_1^{\infty}\frac{(-1)^n}{\sqrt{n}}$, conv. by alt. series test

$\Rightarrow$ Interval $(-7/2, -5/2]$

Peter Townsend

Math 132 Quiz
9 AM - 10 AM

NAME:________________

1. Is the series

$$\sum_{n=2}^{\infty}(-1)^n \frac{3^{2n}}{10^n \cdot \sqrt{n^3+1}} = \sum_{2}^{\infty} \frac{(-1)^n 9^n}{10^n \sqrt{n^3+1}}$$

absolutely convergent, is it conditionally convergent, or is it divergent?

$$L = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{\frac{9^{n+1}}{10^{n+1}\sqrt{(n+1)^3+1}}}{\frac{9^n}{10^n\sqrt{n^3+1}}} = \lim_{n\to\infty} \frac{9}{10} \cdot \frac{1}{\sqrt{\frac{(n+1)^3+1}{n^3+1}}} = \frac{9}{10}$$

So converges absolutely by ratio test

2. What is the interval of convergence of

$$\sum_{n=2}^{\infty} \frac{2^n(-5-x)^n}{\ln(n^3)}? = \sum_{2}^{\infty} \frac{(-1)^n 2^n (x+5)^n}{3\ln(n)}, \quad a_n = \frac{(-1)^n 2^n}{3\ln n}$$

$$R = \lim_{n\to\infty} \left|\frac{a_n}{a_{n+1}}\right| = \lim_{n\to\infty} \frac{\frac{2^n}{3\ln n}}{\frac{2^{n+1}}{3\ln(n+1)}} = \lim_{n\to\infty} \frac{1}{2} \cdot \frac{\ln(n)}{\ln(n+1)} = \frac{1}{2}\left(\lim_{n\to\infty} \frac{\frac{1}{n}}{\frac{1}{n+1}}\right) = \frac{1}{2}\left(\lim_{n\to\infty} \frac{n+1}{n}\right) = \frac{1}{2}$$

$\Rightarrow$ conv. for $|x+5| < 1/2$, or $(-5-1/2, -5+1/2) = (\frac{-11}{2}, \frac{-9}{2})$

Check end points?

$x = \frac{-11}{2} \Rightarrow \sum \frac{(-1)^n 2^n (-1/2)^n}{3\ln n} = \sum \frac{1}{3\ln n}$, div by comp. w/ $b_n = 1/n$

$x = \frac{-9}{2} \Rightarrow \sum \frac{(-1)^n 2^n (1/2)^n}{3\ln n} = \sum \frac{(-1)^n}{3\ln n}$, conv by alt. series test

$\Rightarrow$ Interval $[-11/2, -9/2]$

Peter Townsend

Math 132 Quiz
Noon - 1 PM

NAME:________________

1. Is the series
$$\sum_{n=1}^{\infty}(-1)^n \frac{3^{3n}}{7^{2n}\cdot(n^{1/2}+1)} = \sum_{1}^{\infty} \frac{(-1)^n (27)^n}{(49)^n (n^{1/2}+1)}$$
absolutely convergent, is it conditionally convergent, or is it divergent?

$$L = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{\dfrac{27^{n+1}}{49^{n+1}((n+1)^{1/2}+1)}}{\dfrac{27^n}{49^n(n^{1/2}+1)}} = \frac{27}{49}\lim_{n\to\infty}\frac{\sqrt{n}+1}{\sqrt{n+1}+1} = \frac{27}{49}\lim_{n\to\infty}\frac{\frac{1}{2\sqrt{n}}}{\frac{1}{2\sqrt{n+1}}} = \frac{27}{49}\lim_{n\to\infty}\sqrt{\frac{n+1}{n}}$$
$$= \frac{27}{49} < 1$$

So conv. abs by ratio test

2. What is the interval of convergence of
$$\sum_{n=0}^{\infty}\frac{3^n(-3-x)^n}{\sqrt{1+n^{2/3}}}? = \sum_{0}^{\infty}\frac{(-1)^n 3^n (x+3)^n}{\sqrt{1+n^{2/3}}}, \quad a_n = \frac{(-1)^n 3^n}{\sqrt{1+n^{2/3}}}$$

$$R = \lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right| = \lim_{n\to\infty}\frac{\dfrac{3^n}{\sqrt{1+n^{2/3}}}}{\dfrac{3^{n+1}}{\sqrt{1+(n+1)^{2/3}}}} = \lim_{n\to\infty}\frac{1}{3}\cdot\frac{\sqrt{1+(n+1)^{2/3}}}{\sqrt{1+n^{2/3}}} = \frac{1}{3}\cdot 1 = 1/3$$

$\Rightarrow$ conv. for $|x+3| \le 1/3$, or $(-3-1/3, -3+1/3) = (-10/3, -8/3)$

Check End pts:

$x = -\frac{10}{3} \Rightarrow \sum_{1}^{\infty}\frac{(-1)^n 3^n(-1/3)^n}{\sqrt{1+n^{2/3}}} = \sum_{1}^{\infty}\frac{1}{\sqrt{1+n^{2/3}}}$, div. by limit comparison with $b_n = \frac{1}{n^{1/3}}$

$x = -\frac{8}{3} \Rightarrow \sum_{1}^{\infty}\frac{(-1)^n 3^n(1/3)^n}{\sqrt{1+n^{2/3}}} = \sum_{1}^{\infty}\frac{(-1)^n}{\sqrt{1+n^{2/3}}}$, conv. by alt series

$\Rightarrow$ Interval $(-10/3, -8/3]$