Math 132
Fall 2007 Final Exam

1. Calculate $\int_0^{\frac{\pi}{2}} \cos(x)\sin(x)^3\,dx$.

a) 1 **b)** $\frac{1}{2}$ **c)** $\frac{1}{3}$ **d)** $\frac{1}{4}$ **e)** $\frac{1}{5}$

f) $\frac{2}{3}$ **g)** $\frac{3}{4}$ **h)** $\frac{3}{2}$ **i)** $\frac{4}{3}$ **j)** $\frac{1}{6}$

2. Let $F(x) = \int_x^2 \frac{5 + t^4}{\sqrt{1 + t^3}}\, dt$. **Calculate the derivative D(F)(2) of F at 2.**

a) 4 **b) 5** **c) 6** **d) 7** **e) 8**

f) –4 **g) –5** **h) –6** **i) –7** **j) –8**

3. Calculate $\displaystyle\int_0^1 \frac{x}{(x+1)(x+2)}\,dx.$

a) $\ln\left(\frac{9}{8}\right)$ **b)** $\ln\left(\frac{7}{6}\right)$ **c)** $\ln\left(\frac{5}{4}\right)$ **d)** $\ln\left(\frac{4}{3}\right)$ **e)** $\ln\left(\frac{3}{2}\right)$

f) $\ln\left(\frac{9}{5}\right)$ **g)** $\ln\left(\frac{8}{3}\right)$ **h)** $\ln\left(\frac{9}{4}\right)$ **i)** $\ln\left(\frac{16}{3}\right)$ **j)** $\ln\left(\frac{16}{9}\right)$

4. Calculate $\int_0^1 \frac{8x^2+2x+6}{(1+x)(1+x^2)}\,dx.$

a) $\frac{1}{4}\ln(2)$ **b)** $\frac{1}{2}\ln(2)$ **c)** $\ln(2)$ **d)** $2\ln(2)$ **e)** $3\ln(2)$

f) $4\ln(2)$ **g)** $5\ln(2)$ **h)** $6\ln(2)$ **i)** $7\ln(2)$ **j)** $8\ln(2)$

5. Calculate $\int_{1}^{e} x^2 \ln(x)\, dx$.

a) $\frac{1}{3}\mathbf{e}^3$ **b)** $\frac{1}{3}\left(2\,\mathbf{e}^3 - 1\right)$ **c)** $\frac{1}{3}\left(\mathbf{e}^3 - 2\right)$ **d)** $\frac{2}{3}\left(\mathbf{e}^3 - 1\right)$ **e)** $\frac{1}{3}\left(2\,\mathbf{e}^3 + 1\right)$

f) $\frac{1}{3}\left(\mathbf{e}^3 + 2\right)$ **g)** $\frac{2}{3}\left(\mathbf{e}^3 + 1\right)$ **h)** $\frac{1}{9}\left(2\,\mathbf{e}^3 + 1\right)$ **i)** $\frac{1}{9}\left(\mathbf{e}^3 + 2\right)$ **j)** $\frac{2}{9}\left(\mathbf{e}^3 + 1\right)$

6. What is the derivative of $x^{\left(\frac{1}{x}\right)}$ with respect to x at $x = \frac{1}{2}$?

a) $-\ln(2)$ b) $-\frac{1}{2}\ln(2)$ c) $1-\ln(2)$ d) $1-\frac{1}{2}\ln(2)$ e) $\ln(2)$

f) $\frac{1}{2}\ln(2)$ g) $1+\ln(2)$ h) $1+\frac{1}{2}\ln(2)$ i) $\frac{1}{4}\ln(2)$ j) $\frac{1}{4}$

7. If $y(0) = 0$ **and** $\frac{dy}{dx} = \cos(x)\sqrt{1 - y^2}$, **then what is** $y(x)$?

a) $\sin(\pi \cos(x))$ **b)** $\sin(\sin(x))$ **c)** $\cos\left(\frac{\pi \cos(x)}{2}\right)$ **d)** $\cos(\sin(x)) - 1$ **e)** $\arcsin(x^2)$

f) $\arcsin(\arcsin(x))$ **g)** $\sin(\tan(x))$ **h)** $\tan(\sin(x))$ **i)** $\arcsin(\tan(x))$ **j)** $\arcsin(\arctan(x))$

8. Consider the following three statements about a series $\sum_{n=1}^{\infty} a_n$ with positive terms:

I: The series converges because $\lim_{n \to \infty} a_n = 0$.

II: The series converges because $\lim_{n \to \infty} \frac{a_{n+1}}{b_n} = 1.1$ and $\sum_{n=1}^{\infty} b_n$ converges.

III: The series converges because $\lim_{n \to \infty} \frac{a_{n+1}}{a_n} = 1$.

For each statement, determine whether the reasoning is correct or incorrect.

a) I: correct, II: correct, III: correct
b) I: correct, II: correct, III: incorrect
c) I: correct, II: incorrect, III: correct
d) I: correct, II: incorrect, III: incorrect
e) I: incorrect, II: correct, III: correct
f) I: incorrect, II: correct, III: incorrect
g) I: incorrect, II: incorrect, III: correct
h) I: incorrect, II: incorrect, III: incorrect
i) Wrong answer
j) Bonus wrong answer

9. Consider the following three statements about a series $\sum_{n=1}^{\infty} a_n$ with positive terms:

I: The series converges because $a_n < \dfrac{1}{10+\sqrt{n}}$.

II: The series diverges because $\dfrac{1}{n^2} < a_n$.

III: The series converges because $\lim_{n \to \infty} \dfrac{a_{n+1}}{a_n} = 0$.

For each statement, determine whether the reasoning is correct (C) or incorrect (F).

a) I: C, II: C, III: C
b) I: C, II: C, III: F
c) I: C, II: F, III: C
d) I: C, II: F, III: F
e) I: F, II: C, III: C
f) I: F, II: C, III: F
g) I: F, II: F, III: C
h) I: F, II: F, III: F
i) Wrong answer
j) Bonus wrong answer

10. Consider the three series

I: $\sum_{n=0}^{\infty} \frac{n^5}{3^n}$, **II:** $\sum_{n=0}^{\infty} \frac{10^n}{\sqrt{n!}}$, **and** **III:** $\sum_{n=2}^{\infty} \frac{1}{n \ln(n)}$

and the statements

(C) The series converges
(D) The series diverges

For each series, decide which of statements (C), (D) is correct.

a) I: C, II: C, III: C
b) I: C, II: C, III: D
c) I: C, II: D, III: C
d) I: C, II: D, III: D
e) I: D, II: C, III: C
f) I: D, II: C, III: D
g) I: D, II: D, III: C
h) I: D, II: D, III: D
i) Wrong answer
j) Bonus wrong answer

11. Consider the two series

$$\text{I: } \sum_{n=1}^{\infty} (-1)^n \left(\frac{n}{1+n}\right)^n \quad \text{and} \quad \text{II: } \sum_{n=0}^{\infty} \frac{(-1)^n n^{\left(\frac{2}{3}\right)}}{1+n^{\left(\frac{4}{3}\right)}}$$

and the statements

(AC) The series converges absolutely
(CC) The series converges conditionally
(D) The series diverges

For each series, decide which of statements (AC), (CC), (D) is correct.

a) I: AC, II: AC
b) I: AC, II: CC
c) I: AC, II: D
d) I: CC, II: AC
e) I: CC, II: CC
f) I: CC, II: D
g) I: D, II: AC
h) I: D, II: CC
i) I: D, II: D
j) Wrong answer

12. Consider the two series

$$\text{I:}\ \sum_{n=0}^{\infty} \frac{1}{n^{\pi}} \qquad \text{and} \qquad \text{II:}\ \sum_{n=0}^{\infty} \frac{n!}{10^{(100\,n)}}$$

and the statements

(C) The Ratio Test establishes convergence
(D) The Ratio Test establishes divergence
(F) The Ratio Test is not conclusive.

Apply the Ratio Test to series I and II and for each, decide which of statements (C), (D), (F) is correct.

a) I: C, II: C
b) I: C, II: D
c) I: C, II: F
d) I: D, II: C
e) I: D, II: D
f) I: D, II: F
g) I: F, II: C
h) I: F, II: D
i) I: F, II: F
j) Wrong answer

13. Consider the two series

$$\text{I:}\ \sum_{n=0}^{\infty}\left(\frac{1+n^3}{10+100\,n^2+n^3}\right)^n \qquad \text{and} \qquad \text{II:}\ \sum_{n=1}^{\infty}\left(\frac{3+n}{3\,n}\right)^n$$

and the statements

(C) The Root Test establishes convergence
(D) The Root Test establishes divergence
(F) The Root Test is not conclusive.

Apply the Root Test to series I and II and for each, decide which of statements (C), (D), (F) is correct.

a) I: C, II: C
b) I: C, II: D
c) I: C, II: F
d) I: D, II: C
e) I: D, II: D
f) I: D, II: F
g) I: F, II: C
h) I: F, II: D
i) I: F, II: F
j) Wrong answer

14. Let $f(x) = \frac{1}{72} x^3 e^{(2x^2)}$ **. What is** $f^{(7)}(0)$ **?**

a) 20 b) 40 c) 60 d) 80 e) 100
f) 120 g) 140 h) 160 i) 180 j) 200

15. Calculate $L = \lim_{x \to 0} \dfrac{120 \sin(2x^5)}{x \cos(5x^2) - x}$ **by finding the Maclaurin series of the numerator and the Maclaurin series of the denominator. These two Maclaurin series begin with the same degree p monomial. (In other words, for the Maclaurin series for both the numerator and denominator, the coefficients of x^n are 0 for $n < p$ and the coefficients of x^p are nonzero.) What is the value of the product $p\,L$?**

a) - 36 **b) - 48** **c) - 60** **d) - 72** **e) - 84**
f) - 96 **g) -108** **h) -120** **i) - 132** **j) - 144**

16. Calculate the interval of convergence of $\sum_{n=0}^{\infty} \frac{(-1)^n (x+3)^n}{\sqrt{n+1}\ 4^n}$. **Let R be the radius of convergence. You will need to calculate the sum of four integers and it might help to record them as you go.**

Let c be the base point of the power series. (c = _____)

Set ρ = R if R is an integer and -1 otherwise. (ρ = _____)

Set σ = 1 if the left endpoint belongs to the interval of convergence and 0 otherwise. (σ = _____)

Set τ = 3 if the right endpoint belongs to the interval of convergence and 0 otherwise. (τ = _____)

What is the value of $c + \rho + \sigma + \tau$**?**

a) -4 b) -3 c) -2 d) 2 e) 3
f) 4 g) 7 h) 8 i) 10 j) 11

17. Let $T(x)$ be the degree 2 Taylor polynomial of $\ln(x)$ with base point 2. What is $T(3) - \ln(2)$?

a) $\frac{1}{8}$ **b)** $\frac{1}{4}$ **c)** $\frac{3}{8}$ **d)** $\frac{1}{2}$ **e)** $\frac{5}{8}$

f) $\frac{3}{4}$ **g)** $\frac{7}{8}$ **h)** 1 **i)** $\frac{5}{4}$ **j)** $\frac{3}{2}$

18. To approximate $\int_0^{\frac{1}{2}} \frac{\arctan(x) - x}{x^2}\,dx$, **the Maclaurin series of** $\arctan(x)$ **(and, from that, the Maclaurin series of the integrand) is used. An alternating series for the (exact) value** S **of the definite integral results. An approximation to** S **is obtained by using the minimum number of terms that, by the Alternating Series Test, guarantee an absolute error less than 0.001. What is the approximation?**

a) $-\frac{96}{2401}$ b) $-\frac{209}{5376}$ c) $-\frac{19}{480}$ d) $-\frac{13}{336}$ e) $-\frac{2089}{53760}$

f) $-\frac{25069}{645120}$ g) $-\frac{131}{3360}$ h) $-\frac{523}{13440}$ i) $-\frac{3}{80}$ j) $-\frac{37}{960}$

19. What is the coefficient of x^5 in the Maclaurin series of $\dfrac{8x}{4-x^2}$?

a) $\dfrac{1}{16}$ b) $\dfrac{-1}{16}$ c) $\dfrac{1}{8}$ d) $-\dfrac{1}{8}$ e) $\dfrac{1}{4}$

f) $-\dfrac{1}{4}$ g) $\dfrac{1}{2}$ h) $-\dfrac{1}{2}$ i) 2 j) –2

20. What is the coefficient of x^4 in the Maclaurin series of $\frac{1}{(1+x^2)^{\left(\frac{1}{3}\right)}}$?

a) $-\frac{1}{9}$ b) $\frac{1}{9}$ c) $-\frac{1}{6}$ d) $\frac{1}{6}$ e) $-\frac{2}{9}$

f) $\frac{2}{9}$ g) $-\frac{1}{3}$ h) $\frac{1}{3}$ i) $-\frac{2}{3}$ j) $\frac{2}{3}$