

Math 132
Fall 2007 Exam III

- 1.** The region in the first quadrant that is bounded above by $y = 4x - x^2$ and bounded below by $y = x$ is rotated about the vertical line $x = -1$. What is the volume of the resulting solid of revolution?

- a) $\frac{15\pi}{2}$ b) 16π c) 18π d) 21π e) $\frac{45\pi}{2}$
f) 20π g) $\frac{27\pi}{2}$ h) $\frac{25\pi}{2}$ i) 14π j) 15π

2. The region in the first quadrant bounded above by $y = x$ and below by $y = x^2$ for $0 \leq x \leq 1$ is rotated about the line $y = 2$.

What is the volume of the solid of revolution that results?

- a) $\frac{8\pi}{15}$ b) $\frac{3\pi}{5}$ c) $\frac{2\pi}{3}$ d) $\frac{4\pi}{5}$ e) $\frac{14\pi}{15}$
f) $\frac{\pi}{2}$ g) $\frac{3\pi}{8}$ h) $\frac{5\pi}{8}$ i) $\frac{3\pi}{4}$ j) $\frac{7\pi}{8}$

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- 3. Calculate the arc length of the graph of $y = \frac{x^3}{3} + \frac{1}{4x}$ for $\frac{1}{2} \leq x \leq 1$.

- a) $\frac{5}{12}$
- b) $\frac{11}{24}$
- c) $\frac{1}{2}$
- d) $\frac{13}{24}$
- e) $\frac{7}{12}$
- f) $\frac{5}{8}$
- g) $\frac{2}{3}$
- h) $\frac{17}{24}$
- i) $\frac{3}{4}$
- j) $\frac{5}{6}$



4. The line segment with end points (2,3) and (6,0) is rotated about the y-axis. What is the surface area of the resulting conical frustum?

- a) 12π
- b) 16π
- c) 20π
- d) 24π
- e) 25π
- f) 27π
- g) 28π
- h) 32π
- i) 36π
- j) 40π



5. The graph of $y = 3\sqrt{x}$ for $4 \leq x \leq 10$ is rotated about the x-axis. What is the surface area of the resulting figure?

- a) 54π b) 60π c) 64π d) 72π e) 80π
f) 84π g) 94π h) 102π i) 109π j) 115π

6. Let R be the trapezoidal region in the first quadrant that is bounded above by the graph of $y = 2x$, below by the x -axis, on the left by the vertical line $x = 1$, and on the right by the vertical line $x = 2$. What is the moment of R about the vertical line $x = -1$? (Assume that R has a constant mass density equal to 1.)

a) $\frac{20}{3}$

b) 7

c) $\frac{23}{3}$

d) 8

e) $\frac{25}{3}$

f) 9

g) $\frac{32}{3}$

h) 11

i) $\frac{35}{3}$

j) 12

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7. Let R be the region in the first quadrant that is bounded above by the graph of $y = 4 - x^2$ and below by the x -axis. What is the x -coordinate of the center of mass of R ?

- a) $\frac{1}{3}$
- b) $\frac{3}{8}$
- c) $\frac{2}{5}$
- d) $\frac{1}{2}$
- e) $\frac{3}{5}$
- f) $\frac{5}{8}$
- g) $\frac{2}{3}$
- h) $\frac{3}{4}$
- i) $\frac{5}{4}$
- j) $\frac{3}{2}$

8. What is the y-coordinate of the center of mass of the region **R** of the preceding question?

- a) $\frac{5}{3}$ b) $\frac{6}{5}$ c) $\frac{11}{8}$ d) $\frac{4}{3}$ e) $\frac{7}{5}$
f) $\frac{13}{8}$ g) $\frac{8}{5}$ h) $\frac{7}{4}$ i) $\frac{9}{5}$ j) $\frac{15}{8}$

9. Suppose that $f(x) = x^2$. If $f(7)$ is equal to the average value of $f(x)$ over the interval $[2, b]$, then what is b ?

- a) 8 b) $\frac{25}{3}$ c) 9 d) $\frac{29}{3}$ e) 10
f) $\frac{31}{3}$ g) 11 h) $\frac{35}{3}$ i) 12 j) $\frac{37}{3}$



10. Suppose that $f(x) = \frac{c}{1+x^2}$ is the probability density function of a random variable that has values in the interval $[0, \sqrt{3}]$. What is c ?

- a) $\frac{1}{\pi}$ b) $\frac{2}{\pi}$ c) $\frac{3}{\pi}$ d) $\frac{4}{\pi}$ e) $\frac{5}{\pi}$
f) π g) $\frac{\pi}{2}$ h) $\frac{\pi}{3}$ i) $\frac{\pi}{4}$ j) $\frac{\pi}{5}$

11. A random variable X that assumes values in the interval $[0, 1]$ has probability density function

$$f(x) = \frac{8}{(\pi + 2)(1 + x^2)^2} \quad \text{for } 0 \leq x \leq 1.$$

If μ_X is the mean of X , what is $(\pi + 2)\mu_X$?

- a) 2 b) $\sqrt{2}$ c) 3 d) $\sqrt{3}$ e) 5
f) $\sqrt{5}$ g) 6 h) $\sqrt{6}$ i) $2\sqrt{2}$ j) $2\sqrt{3}$



12. A random real number X is chosen from the interval $[1, 9]$. If the p.d.f. of X is

$f(x) = \frac{3\sqrt{x}}{52}$, then what is the probability that X is greater than 4 ?

- a) $\frac{7}{13}$ b) $\frac{15}{26}$ c) $\frac{8}{13}$ d) $\frac{17}{26}$ e) $\frac{9}{13}$
f) $\frac{19}{26}$ g) $\frac{10}{13}$ h) $\frac{21}{26}$ i) $\frac{11}{13}$ j) $\frac{23}{26}$

13. A random variable X has pdf given by $f(x) = \frac{2x+1}{108}$ for $1 \leq x \leq 10$.

What is the median m of X ?

- a) 5 b) $\frac{16}{3}$ c) $\frac{17}{3}$ d) 6 e) $\frac{13}{2}$
f) 7 g) $\frac{22}{3}$ h) $\frac{23}{3}$ i) 8 j) $\frac{17}{2}$

14. Starting from equilibrium, 60 J of work are expended when a spring is stretched 2 meters. How many meters beyond equilibrium can the spring be stretched with a force of 45 N?

- a) $\frac{1}{5}$ b) $\frac{1}{2}$ c) 1 d) $\frac{6}{5}$ e) $\frac{3}{2}$
f) $\frac{8}{5}$ g) 2 h) $\frac{12}{5}$ i) $\frac{5}{2}$ j) 3

15. The vertical cross-sections of a tank have the shape $y = |x| - 12$ for $-12 \leq x \leq 12$ (measured in meters). Horizontal cross-sections of the tank are rectangles that are 5 m long. The widths of these rectangles vary, but can be deduced from the description of the vertical cross-sections. The tank is filled to the top with a fluid that has weight density equal to $\frac{3}{10} \frac{N}{m^3}$. If after pumping water to the top of the tank, the remaining water is 10 m deep, how many Joules of work have been done?

- a) 24 b) 32 c) 36 d) 48 e) 52
f) 56 g) 64 h) 72 i) 80 j) 84

16. Forty feet of a uniform cable hang over the side of a building. The cable weighs 2 lbs/ft. A 48 pound hook is attached to the dangling end of the cable. The cable is pulled up 30 feet before the hook snags on a balcony rail. How many foot-pounds of work have been done?

- a) 2480 b) 2540 c) 2580 d) 2640 e) 2680
f) 2740 g) 2800 h) 2840 i) 2880 j) 2940



17. Calculate

$$\int_7^8 \frac{1}{(8-x)^{\left(\frac{1}{3}\right)}} dx.$$

a) $\frac{4}{3}$

b) $\frac{3}{2}$

c) $\frac{5}{3}$

d) 2

e) $\frac{7}{3}$

f) $\frac{5}{2}$

g) $\frac{8}{3}$

h) 3

i) $\frac{10}{3}$

j) $\frac{7}{2}$

18.

Calculate

$$\int_1^{\infty} \frac{x^2}{(2+x^3)^2} dx.$$

- a) 1 b) $\frac{1}{2}$ c) $\frac{1}{3}$ d) $\frac{1}{4}$ e) $\frac{1}{5}$
f) $\frac{1}{6}$ g) $\frac{1}{7}$ h) $\frac{1}{8}$ i) $\frac{1}{9}$ j) $\frac{1}{10}$

- 19. Calculate the 100th partial sum of $\sum_{n=1}^{\infty} \frac{1}{(n+1)n}$.

- a) 1 b) $\frac{99}{100}$ c) $\frac{100}{99}$ d) $\frac{100}{101}$ e) $\frac{101}{100}$
f) $\frac{49}{50}$ g) $\frac{50}{49}$ h) $\frac{98}{101}$ i) $\frac{101}{98}$ j) $\frac{1}{10100}$

20. Consider the series

I) $\sum_{n=1}^{\infty} \frac{2^n}{n}$ II) $\sum_{n=1}^{\infty} \tan\left(\frac{1}{n}\right)$ III) $\sum_{n=1}^{\infty} \sec\left(\frac{1}{n}\right)$ IV) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

List all given series for which the Divergence Test yields a conclusion.

- a) I b) II c) III d) IV e) I, II
f) I, III g) I, IV h) II, III i) II, IV j) III, IV