Math 132
Fall 2007 Exam II

Integral Formula:

$$\int \sec(t)^3\, dt = \frac{1}{2}\sec(t)\tan(t) + \frac{1}{2}\ln(|\sec(t)+\tan(t)|) + C$$

1. Suppose that $f(x) = 2^{\sqrt{x}}$. Calculate $D(f)(9)$, the derivative of $f(x)$ evaluated at x = 9.)

a) $\frac{2}{3}\ln(2)$ b) $\frac{2}{9}\ln(2)$ c) $\frac{4}{3}\ln(2)$ d) $\frac{4}{9}\ln(2)$ e) $\frac{4}{27}\ln(2)$

f) $\frac{2}{3\ln(2)}$ g) $\frac{2}{9\ln(2)}$ h) $\frac{4}{3\ln(2)}$ i) $\frac{4}{9\ln(20)}$ j) $\frac{4}{27\ln(2)}$

2. Calculate $\int_1^3 \log_3(x)\,dx$.

a) $2 - \frac{1}{\ln(3)}$ b) $2 + \frac{1}{\ln(3)}$ c) $2 - \ln(3)$ d) $2 + \ln(3)$ e) $2\ln(3) - 1$

f) $2\ln(3) + 1$ g) $3 - \frac{2}{\ln(3)}$ h) $3 + \frac{2}{\ln(3)}$ i) $1 - \frac{2}{\ln(3)}$ j) $1 + \frac{2}{\ln(3)}$

3. Suppose that $f(x) = x^{\sqrt{x}}$. Calculate D(f)(4). (D(f)(4) is the derivative of f(x) evaluated at x = 4.)

a) $1 + \ln(2)$ **b) $2(2 + \ln(2))$** **c) $4(2 + \ln(2))$** **d) $2(1 + \ln(2))$** **e) $8 + \ln(2)$**

f) $2 + \ln(2)$ **g) $4 + \ln(2)$** **h) $4(1 + \ln(2))$** **i) $2(4 + \ln(2))$** **j) $8(1 + \ln(2))$**

4. A radioactive substance has mass 120g at time $t = 4$ and mass 90g at time $t = 6$. What is the mass at $t = 12$?

a) $\frac{1979}{32}$ **b)** $\frac{1985}{32}$ **c)** $\frac{1991}{32}$ **d)** $\frac{1997}{32}$ **e)** $\frac{1203}{21}$

f) $\frac{1209}{32}$ **g)** $\frac{1215}{32}$ **h)** $\frac{1221}{32}$ **i)** $\frac{1227}{32}$ **j)** $\frac{1233}{32}$

5. The mass of a microbe colony splashing about in a nutrient broth triples every 12 hours. What is the colony's doubling time?

a) 8 b) $8\ln(2)$ c) $8\ln(3)$ d) $\frac{8\ln(2)}{\ln(3)}$ e) $\frac{12\ln(2)}{\ln(3)}$

f) $12\ln\left(\frac{3}{2}\right)$ g) $\frac{3\ln(12)}{2}$ h) $3\ln(2)$ i) $2\ln(3)$ j) $\frac{2\ln(12)}{3}$

6. Suppose that $u(t)$ is the unique solution of the initial value problem

$\frac{d}{dt} u(t) = M - 5\,u(t),\ u(0) = 1$ **where M is a constant.**

If $\lim_{t \to \infty} u(t) = 12$ then what is M?

a) 3 **b) 4** **c) 5** **d) 10** **e) 12**

f) 15 **g) 20** **h) 30** **i) 60** **j) 120**

7. Suppose that $f(x) = \text{arcsec}(5x)$. Calculate $D(f)\left(-\frac{1}{3}\right)$. (The derivative of f(x) at $x = -\frac{1}{3}$).

a) $-\frac{9}{4}$ **b)** $-\frac{5}{4}$ **c)** $-\frac{4}{3}$ **d)** $-\frac{5}{3}$ **e)** $-\frac{9}{20}$

f) $\frac{9}{4}$ **g)** $\frac{5}{4}$ **h)** $\frac{4}{3}$ **i)** $\frac{5}{3}$ **j)** $\frac{9}{20}$

8. Suppose that f(x) = 120 arctan($\sqrt{x}$). What is D(f)(4)? (The derivative of f(x) at x = 4).

a) 2 b) 3 c) 4 d) 5 e) 6
f) 8 g) 10 h) 12 i) 15 j) 20

9. Calculate $\int_0^{\frac{1}{2}} x\, e^{(2x)}\, dx$.

a) $\frac{1}{4}$ **b)** $\frac{1}{2}$ **c)** $\frac{3}{4}$ **d) 1** **e) 2**

f) $\frac{e}{4}$ **g)** $\frac{e}{2}$ **h)** $\frac{3e}{4}$ **i) e** **j) 2 e**

10. Calculate $\int_0^{\pi} x^2 \sin(x)\,dx$.

a) π^2 **b)** $\pi^2 - 1$ **c)** $\pi^2 - 2$ **d)** $\pi^2 - 3$ **e)** $\pi^2 - 4$

f) $\pi^2 + 5$ **g)** $\pi^2 + 4$ **h)** $\pi^2 + 3$ **i)** $\pi^2 + 2$ **j)** $\pi^2 + 1$

11. Calculate $25 \int_{1}^{e} x^4 \ln(x)\, dx$.

a) $e^5 - 1$ **b)** $2e^5 - 1$ **c)** $3e^5 - 1$ **d)** $4e^5 - 1$ **e)** $5e^5 - 1$

f) $e^5 + 1$ **g)** $2e^5 + 1$ **h)** $3e^5 + 1$ **i)** $4e^5 + 1$ **j)** $5e^5 + 1$

12. Calculate $\int_1^2 \frac{5x+2}{x^2+x}\,dx$.

a) $\ln(2)$ **b)** $\ln(3)$ **c)** $2\ln(2)$ **d)** $2\ln(3)$ **e)** $3\ln(2)$

f) $\ln(6)$ **g)** $\ln\left(\frac{2}{3}\right)$ **h)** $\ln\left(\frac{3}{2}\right)$ **i)** $\ln\left(\frac{9}{2}\right)$ **j)** $\ln\left(\frac{27}{2}\right)$

13. Find an ordered triple (α, β, γ) of positive integers α, β, γ such that

$$\int_1^2 \frac{x^2 + 3x - 4}{x\,(x+2)^2}\,dx = \alpha \ln(\beta) - \beta \ln(\alpha) + \frac{1}{\gamma}.$$

a) 2, 3, 4 **b) (3, 2, 4)** **c) (4, 3, 2)** **d) (4, 2, 3)** **e) 3, 4, 2**

f) (3, 2, 3) **g) (2, 3, 2)** **h) (4, 2, 2)** **i) (4, 3, 3)** **j) (3, 2, 2)**

14. Calculate $\int_0^{\pi} \sin(x)^3 \cos(x)^2 \, dx$.

a) 1/15 **b) 2/15** **c) 1/5** **d) 4/15** **e) 1/3**
f) 2/5 **g) 7/15** **h) 8/15** **i) 3/5** **j) 2/3**

15. Use the reduction formula

$$\int_0^1 x^n \, e^{(-x^2)} \, dx = \frac{(n-1)\int_0^1 x^{(n-2)} \, e^{(-x^2)} \, dx}{2} - \frac{1}{2e}$$

and the approximation $\int_0^1 e^{(-x^2)} \, dx \simeq \frac{3}{4}$ **to approximate** $\int_0^1 x^4 \, e^{(-x^2)} \, dx$**.**

a) $\frac{1}{2} - \frac{1}{4e}$ **b)** $\frac{1}{2} - \frac{3}{4e}$ **c)** $\frac{3}{4} - \frac{1}{2e}$ **d)** $\frac{3}{4} - \frac{3}{2e}$ **e)** $\frac{9}{16} - \frac{3}{4e}$

f) $\frac{9}{16} - \frac{5}{4e}$ **g)** $\frac{5}{8} - \frac{5}{4e}$ **h)** $\frac{5}{8} - \frac{3}{4e}$ **i)** $\frac{3}{4} - \frac{1}{e}$ **j)** $\frac{9}{16} - \frac{1}{e}$

16. Calculate $\int_{\frac{1}{\sqrt{2}}}^{1} \sqrt{1-x^2}\,dx$.

a) $\frac{\pi-2}{8}$ b) $\frac{\pi-2}{4}$ c) $\frac{\pi-2}{2}$ d) $\frac{\pi-1}{8}$ e) $\frac{\pi-1}{4}$

f) $\frac{\pi-1}{2}$ g) $\frac{4-\pi}{8}$ h) $\frac{4-\pi}{4}$ i) $\frac{4-\pi}{2}$ j) $\frac{8-\pi}{8}$

17. Calculate $\displaystyle\int_0^1 \frac{x^2}{\sqrt{1+x^2}}\,dx.$

a) $1-\dfrac{\ln(1+\sqrt{2})}{2}$

b) $1+\dfrac{\ln(1+\sqrt{2})}{2}$

c) $\sqrt{2}+\ln(1+\sqrt{2})$

d) $\sqrt{2}-\ln(1+\sqrt{2})$

e) $1-\ln(1+\sqrt{2})$

f) $1+\ln(1+\sqrt{2})$

g) $\dfrac{1}{2}-\ln\left(1+\dfrac{1}{\sqrt{2}}\right)$

h) $\dfrac{1}{2}+\ln\left(1+\dfrac{1}{\sqrt{2}}\right)$

i) $\dfrac{\sqrt{2}}{2}-\dfrac{\ln(1+\sqrt{2})}{2}$

j) $\dfrac{\sqrt{2}}{2}+\dfrac{\ln(1+\sqrt{2})}{2}$

18. Evaluate $\int_0^1 \frac{4x^2 + 2x + 4}{(x^2 + 1)^2}\,dx$.

a) $2\pi - 1$ **b)** $2\pi + 1$ **c)** $\pi - \frac{1}{4}$ **d)** $\pi - \frac{1}{2}$ **e)** $\pi - 1$

f) π **g)** $\pi + \frac{1}{4}$ **h)** $\pi + \frac{1}{2}$ **i)** $\pi + 1$ **j)** 2π

19. The region in the first quadrant bounded above by $y = 2 - x$ and below by $y = \sqrt{x}$ is rotated about the x-axis. What is the volume of the resulting solid of revolution?

a) 2π **b)** $\frac{11\pi}{6}$ **c)** $\frac{5\pi}{3}$ **d)** $\frac{3\pi}{2}$ **e)** $\frac{4\pi}{3}$

f) $\frac{5\pi}{2}$ **g)** $\frac{8\pi}{3}$ **h)** $\frac{9\pi}{4}$ **i)** $\frac{7\pi}{4}$ **j)** $\frac{5\pi}{4}$

20. The region above the x-axis and under the parabola $y = 1 - (x - 2)^2$, $1 < x \leq 3$ is rotated about the y-axis. What is the volume of the resulting solid of revolution?

a) $\frac{5\pi}{3}$ **b)** $\frac{8\pi}{3}$ **c)** $\frac{10\pi}{3}$ **d)** 4π **e)** $\frac{9\pi}{2}$

f) 5π **g)** $\frac{16\pi}{3}$ **h)** $\frac{21\pi}{4}$ **i)** 6π **j)** $\frac{20\pi}{3}$