

Read the Introductory Example for Chapter 1 (pp. 1-2) and Example 1 in Section 1.6 (pp. 49-51)

Suppose an “economy” has 4 sectors only: agriculture (A), Energy (E), manufacturing (M) and transportation (T).

In this simple “closed exchange economy”, all goods are bought and sold among these four sectors.

The production/consumption among the sectors are summarized in an exchange matrix

$$\begin{array}{cccc}
 & \text{Production of sector} & & \\
 & \downarrow & \downarrow & \downarrow & \downarrow \\
 & A & E & M & T \\
 \\
 \left[\begin{array}{cccc}
 .65 & .30 & .30 & .20 \\
 .10 & .10 & .15 & .10 \\
 .25 & .35 & .15 & .30 \\
 0 & .25 & .40 & .40
 \end{array} \right] & \rightarrow \text{consumed by} & \left\{ \begin{array}{l} A \\ E \\ M \\ T \end{array} \right.
 \end{array}$$

Going down a column, we can read off what part of a given sector's production is consumed by each of the other sectors: for example, the production of the manufacturing sector (M) is consumed as

30% to the agriculture sector,
 15% to the energy sector,
 15% to the manufacturing sector (it uses up some of its own goods), and
 40% to the transportation sector.

Therefore, of course, the sum of each column is 1 (100%).

Going across a row, we can read off what part of the production of each sector is consumed by each of the other sectors: for example, the energy sector uses 10% of the production of the agriculture, energy and transportation sectors and 15% of the production of the manufacturing sector.

Suppose p_A, p_E, p_M, p_T represent the total production of goods in each sector; to have a common unit of measurement, let's say these quantities are all measured in \$ (value).

Question: is it possible to set production levels for the four sectors so that “everybody's happy” – that is, every sector gets what it needs (using its own production to “pay”) and nothing is left over. If so, then the values p_A, p_E, p_M, p_T are called equilibrium prices for this “closed exchange economy.”

The “cost” to sector A for what it needs is

$$.65P_A + .30P_E + .30P_M + .20P_T$$

This expense must be “paid for” by the value of sector A 's goods, P_A , so

$$.65P_A + .30P_E + .30P_M + .20P_T = P_A.$$

Similarly, we need

$$\begin{aligned} .10P_A + .10P_E + .15P_M + .10P_T &= P_E \\ .25P_A + .35P_E + .15P_M + .30P_T &= P_M \\ .25P_E + .40P_M + .40P_T &= P_T \end{aligned}$$

Rearranging these equations gives the linear system

$$\begin{aligned} -.35P_A + .30P_E + .30P_M + .20P_T &= 0 \\ .10P_A - .90P_E + .15P_M + .10P_T &= 0 \\ .25P_A + .35P_E - .85P_M + .30P_T &= 0 \\ .25P_E + .40P_M - .60P_T &= 0 \end{aligned}$$

whose augmented matrix is

$$\begin{bmatrix} -.35 & .30 & .30 & .20 & 0 \\ .10 & -.90 & .15 & .10 & 0 \\ .25 & .35 & -.85 & .30 & 0 \\ 0 & .25 & .40 & -.60 & 0 \end{bmatrix}. \quad \text{The row reduced echelon}$$

form (with entries rounded to 2 decimal places for convenience) is

$$\begin{bmatrix} 1 & 0 & 0 & -2.03 & 0 \\ 0 & 1 & 0 & -0.53 & 0 \\ 0 & 0 & 1 & -1.17 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Writing \approx instead of $=$ (since we rounded decimals), we have

$$\begin{aligned} P_A &\approx 2.03P_T \\ P_E &\approx 0.53P_T \\ P_M &\approx 1.17P_T \\ P_T &\text{ free} \end{aligned}$$

Therefore: So if a total production P_T (\$) for the transportation sector is determined, then the productions of the other sectors can be adjusted to create an equilibrium.