

Math 308 - Exam 2 - March 14, 2003

The test contains 8 questions, each of equal value. Whenever possible, answers should be written using exact numbers. For example: write  $\frac{2}{3}$  instead of 0.67,  $\pi$  instead of 3.1415,  $e^2$  instead of 7.4, etc.

1. Evaluate:

$$\left| \frac{2 - 3i}{2 + 3i} \right|$$

2. Express the following complex number in the  $x + iy$  form.

$$e^{(i\pi/4) + (\ln 2)/2}$$

3. Solve the following set of equations:

$$\begin{aligned} 14 - x + 5y &= 0 \\ 7z + 2x - 15 &= 0 \\ x - y + 3z &= 9. \end{aligned}$$

4. Find all values of  $a, b, c$  so that the following is an orthogonal matrix.

$$\begin{pmatrix} 1/2 & 0 & a \\ 0 & 1 & b \\ \sqrt{3}/2 & 0 & c \end{pmatrix}$$

5. Find the cosine of the angle between the two vectors in  $\mathbb{R}^5$ :

$$(2, 0, 4, 6, 5), (-5, 1, 5, 3, -2)$$

6. Find the eigenvalues and the respective eigenvectors of

$$\begin{pmatrix} 2 & 2 \\ 2 & -1 \end{pmatrix}$$

7. Rotate to principal axes the following conic:

$$2x^2 + 4xy - y^2 = 24$$

(Use your result for the previous problem.)

8. Show that the product  $AB$  of two arbitrary  $n \times n$  orthogonal matrices is also an orthogonal matrix. (Recall that a matrix  $C$  is said to be orthogonal if  $C^T = C^{-1}$ .)