Math 5052 - Homework 9

Due 04/02/09

1. (Problem 16, page 224) Suppose that $I \in C_0(X, \mathbb{R})^*$ and I^+, I^- are the functionals constructed in the proof of Lemma 7.15. If μ is the signed Radon measure associated to I, then the positive and negative variations of μ are the Radon measures associated to I^+ and I^-.

2. (Problem 17, page 224) If μ is a positive Radon measure on X with $\mu(X) = \infty$, there exists $f \in C_0(X)$ such that $\int f \, d\mu = \infty$. Consequently, every positive linear functional on $C_0(X)$ is bounded.

3. (Problem 19, page 225) Let X be a completely regular space and $\mathcal{A}$ a completely regular subalgebra of $BC(X)$ (see Exercise 73 in §4.8). Find a description of $\mathcal{A}^*$ as a space of measures.

4. (Problem 24, page 225) Find examples of sequences $\{\mu_n\}$ in $M(\mathbb{R})$ such that

   (a) $\mu_n \to 0$ vaguely, but $\|\mu_n\| \nrightarrow 0$.

   (b) $\mu_n \to 0$ vaguely, but $\int f \, d\mu_n \nrightarrow 0$ for some bounded measurable f with compact support.

   (c) $\mu_n \geq 0$ and $\mu_n \to 0$ vaguely, but there exists $x \in \mathbb{R}$ such that $F_n(x) \nrightarrow 0$ (notation as in Proposition 7.19).

5. (Problem 30, page 231) Let μ and ν be Radon measures on X and Y, not necessarily σ-finite. If f is a nonnegative LSC function on $X \times Y$, then $x \mapsto \int f_x \, d\nu$ and $y \mapsto \int f^y \, d\mu$ are Borel measurable and $\int f \, d(\mu \hat{\times} \nu) = \iint f \, d\mu \, d\nu = \iint f \, d\nu \, d\mu$.