Math 5031 - Homework 3

Due 9/23/05

1. Let p be prime and let G be a p-group, that is, a group of order p^k for some positive integer k. Let A be a normal subgroup of G of order p. Prove that A is contained in the center of G.

2. Let p be a prime number. Show that a group of order p^2 is abelian, and that there are only two such groups up to isomorphism.

3. Let G be a group of order p^3, where p is prime, and G is not abelian. Let Z be its center. Let C be a cyclic group of order p. Show that Z is isomorphic to C and G/Z is isomorphic to the direct product $C \times C$.

4. Let G be a group of order pq, where p and q are prime and $p < q$. Assume that q is not equivalent to 1 modulo p. Prove that G is cyclic.

5. We call P a p-*Sylow* subgroup of a finite group G if the order of P is p^n and p^n is the highest power of p dividing the order of G. Let H be a normal subgroup of order p of a finite group G. Prove that H is contained in every p-Sylow subgroup of G.