

# Math 350 - Homework 2

Due 2/05/2010

1. (Text, problem 7, page 35.) If  $X$  and  $Y$  have a joint probability density function given by

$$f(x, y) = 2e^{-(x+2y)}$$

for  $x$  and  $y$  in  $(0, \infty)$ , find the probability  $P(X < Y)$ .

2. (Text, problem 9, page 36.) The continuous random variable  $X$  has a probability density function given by

$$f(x) = cx$$

for  $0 < x < 1$ . Find the expected value  $E[X]$ . (You need to determine the value of  $c$ .)

3. (Text, problem 15, page 36.) An airplane needs at least half of its engines to safely complete its mission. If each engine independently functions with probability  $p$ , for what values of  $p$  is a three-engine plane safer than a five-engine plane? (It may be necessary to find the roots of a cubic polynomial. It may be helpful to plot the graph of the polynomial over the interval  $[0, 1]$  using, say, Matlab or Scilab.)

4. (Text, problem 19, page 37.) If  $X$  is a Poisson random variable with parameter  $\lambda$ , show that

(a)  $E[X] = \lambda$ .

(b)  $\text{Var}(X) = \lambda$ .

5. (Text, problem 23, page 37.) Two players play a certain game until one has won a total of five games. If player  $A$  wins each individual game with probability 0.6, determine:

(a) what is the probability she will win the match?

(b) what is the expected number of games in a match?

(c) Confirm your result by doing a computer simulation of the situation. For example, you can simulate the outcome of a single game by flipping a biased coin (as in homework 1) with probability of heads equal to 0.6. Play the coin game a number of times, keeping a record of the accumulated number of heads and tails. The process stops the first time that the count of heads or tails reaches 5. (This cannot take more than 9 tosses.) Now determine which one (heads or tail) reached 5 first, and how many steps it took for that to happen. By repeating the process a large number of times (say, 1000) count the frequency of the times when heads (the first player) wins. Similarly, obtain the average number of games (coin tosses) in a match.

**I plan to discuss some of these problems in class during the week. It will help a lot if you think about the problems, especially problem number 5, before I comment on them.**