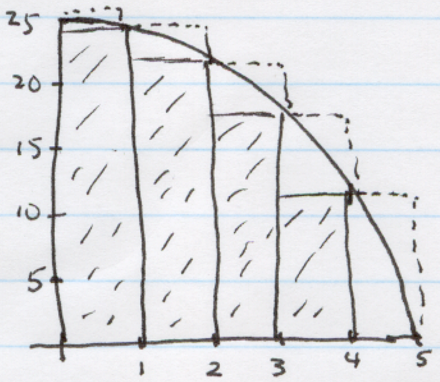
MATH 1323 – SOLUTIONS FOR HOMEWORK #1

1 §5.1, p. 355, # 4

x	$f(x)=25-x^2$
0	25
1	24
2	21
3	16
4	9
5	0



(a) R_5 = shaded areas $= \{f(1)+f(2)+f(3)+f(4)+f(5)\}\,\underset{\Delta x}{1}$

$= \boxed{70}$

(b) L_5 = areas under dotted lines $= \{f(0)+f(1)+f(2)+f(3)+f(4)\}\,1$

$= \boxed{95}$

2 §5.1, p. 356, #12

i	t_i	$\Delta t_i = t_i - t_{i-1}$	$v(t_i)$
0	0	—	0
1	10	10	185
2	15	5	319
3	20	5	447
4	32	12	742
5	59	27	1325
6	62	3	1445
7	125	63	4151

Available Riemann sum approximations to $h(62)$ = height in feet 62 seconds after lift off $= \int_0^{62} v(t)\,dt$

$L_6 = \sum_{i=1}^{6} v(t_{i-1})\Delta t_i = \boxed{31,893}$

$R_6 = \sum_{i=1}^{6} v(t_i)\Delta t_i = \boxed{54,694}$

$T_6 = \sum_{i=1}^{6} \left[\frac{v(t_{i-1})+v(t_i)}{2}\right]\Delta t_i = \frac{L_6+R_6}{2}$

$= \boxed{43,293.5}$

T_6 is the <u>best</u> available approximation.

③ $\boxed{\S\, 5.1,\ p.356,\ \#\ 18}$

(a) $\int_0^1 x^3\,dx = \lim_{n\to\infty} R_n = \lim_{n\to\infty} \sum_{i=1}^{n} (i/n)^3 \frac{1}{n}$

(b) Using $1^3 + 2^3 + \cdots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$

$$R_n = \left[\frac{n(n+1)}{2}\right]^2 \frac{1}{n^3}\,\frac{1}{n} = \frac{1}{4}\left(\frac{n+1}{n}\right)\left(\frac{n+1}{n}\right)$$

Since $\frac{n+1}{n} = (1 + 1/n) \to 1$ as $n \to \infty$

$$\lim_{n\to\infty} R_n = \frac{1}{4}\cdot 1\cdot 1 = \boxed{\frac{1}{4}}$$

④ $\boxed{\S\, 5.2,\ p.367,\ \#6}$

From the graph in the text we get the following table

x	g(x)
-3	2
-2.5	1.6
-2	1
-1.5	0
-1	-0.5
-0.5	-1
0	-1.5
0.5	-1.8
1	-1.5
1.5	-1
2	-0.5
2.5	0.5
3	2.5

a) $R_6 = \{g(-2) + g(-1) + g(0) + g(1) + g(2) + g(3)\}\,1$
$= \boxed{-0.5}$

(b) $L_6 = \{g(-3) + g(-2) + g(-1) + g(0) + g(1) + g(2)\}\,1$
$= \boxed{-1}$

(c) $M_6 = \{g(-2.5) + g(-1.5) + g(-0.5) + g(0.5)$
$+ g(1.5) + g(2.5)\}\,1$
$= \boxed{-1.7}$

$1.6 + 0 + (-1)$

(5) $\boxed{\S\, 5.2,\ p.367,\ \#8}$

x	f(x)
0	9.3
1	9.0
2	8.3
3	6.5
4	2.3
5	-7.6
6	-10.5

We estimate $\int_0^6 f(x)dx$ using $\Delta x = \frac{6}{3} = 2$ and the 3 intervals $[0,2]$, $[2,4]$, $[4,6]$

a) $R_3 = \{f(2) + f(4) + f(6)\}2$
$= \{8.3 + 2.3 - 10.5\}2 = \boxed{.2}$

b) $L_3 = \{f(0) + f(2) + f(4)\}2$
$= \{9.3 + 8.3 + 2.3\}2 = \boxed{39.8}$

c) $M_3 = \{f(1) + f(3) + f(5)\}2$
$= \{9.0 + 6.5 - 7.6\}2 = \boxed{15.8}$

We're uncertain whether M_3 is bigger or smaller than $\int_0^6 f(x)dx$
However, because f is decreasing on $[0,6]$, $L_3 \geq \int_0^6 f(x)dx \geq R_3$

(6) $\boxed{\S\, 5.2,\ p.368,\ \#22}$ Compute $\int_1^5 (2 + 3x - x^2)dx = \lim_{n\to\infty} R_n$

For R_n, we use $\Delta x = \frac{5-1}{4} = 4/n$ and
$x_i = 1 + i\Delta x = 1 + \frac{4i}{n}$

so $R_n = \sum_{i=1}^{n} \left\{2 + 3(1 + \frac{4i}{n}) - (1 + \frac{4i}{n})^2\right\} \frac{4}{n}$

$$= \sum_{i=1}^{n} \left\{4 \cdot \frac{4}{n} + \frac{4}{n}\frac{4}{n} i - \left(\frac{4}{n}\right)^3 i^2\right\}$$

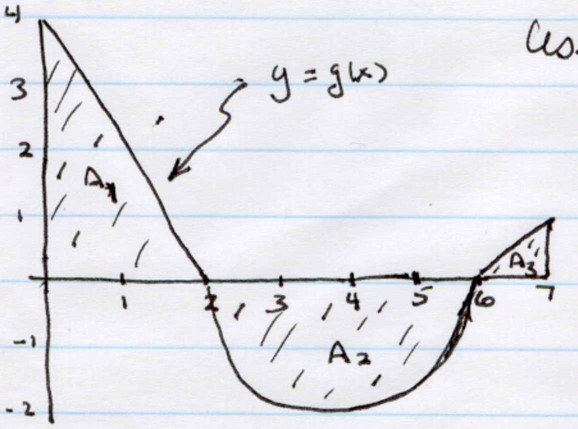
$$= \frac{16}{n} \cdot n + \frac{16}{n^2}\frac{n(n+1)}{2} - \frac{64}{n^3}\frac{n(n+1)(2n+1)}{6}$$

$$= 16 + 8 \cdot 1 \cdot (1 + 1/n) - \frac{64}{6} \cdot 1 \cdot (1 + 1/n)(2 - \frac{1}{n})$$

As $n \to \infty$, $\frac{1}{n} \to 0$ so

$$\int_1^5 (2 + 3x - x^2)\,dx = 16 + 8 - \frac{64}{6} \cdot 2 = 24 - \frac{64}{3}$$
$$= \boxed{8/3}$$

⑦ $\boxed{\S 5.2,\ p.368,\ \#30}$



Using standard formulas for areas of triangles and circles,

(a) $\int_0^2 g(x)\,dx = A_1$

$= \frac{1}{2}(2)(4) = \boxed{4}$

(b) $\int_2^6 g(x)\,dx = -A_2$

$= -\frac{1}{2}\pi(2)^4 = \boxed{-2\pi}$

(c) $\int_0^7 g(x)dx = A_1 - A_2 + A_3$

$= 4 - 2\pi + \frac{1}{2}(1)(1)$

$= \boxed{\frac{9}{2} - 2\pi}$

MATH 1323

SOLUTIONS – HOMEWORK # 2

① [modification of # 14, p. 368]

(a) We wish to use the TI-83 to compute for $n=10$ and $n=100$ the Riemann sum approximations L_n, R_n, T_n, and M_n for the integral $I = \int_1^2 \sqrt{1+x^2}\,dx$. The definitions are

$$L_n = \left(\sum_{i=1}^{n} y_{i-1}\right) 1/n$$
$$R_n = \left(\sum_{i=1}^{n} y_i\right) 1/n$$
$$T_n = \left(\sum_{i=1}^{n} \frac{(y_{i-1}+y_i)}{2}\right) 1/n = \tfrac{1}{2}(L_n+R_n)$$
$$M_n = \left(\sum_{i=1}^{n} \bar{y}_i\right) 1/n$$

where $y_i = \sqrt{1+x_i^2} = \sqrt{1+(1+i/n)^2}$

and $\bar{y}_i = \sqrt{1+\bar{x}_i^2} = \sqrt{1+(1+i/n-1/2n)^2}$

since $\bar{x}_i = \frac{1}{2}(x_{i-1}+x_i) = \frac{1}{2}\left(1+\frac{i-1}{n}\right)+\frac{1}{2}(1+i/n)$
$= 1 + i/n - 1/2n$.

(i) We can create in list L_2 the sequence of y_i values by the command $L_2 = \text{seq}(\sqrt{(1+(1+I/n)^\wedge 2)}, I, 1, n)$

Then $\text{sum}(L_2) \div n$ produces the R_n estimate

(ii) $L_1 = L_2$ copies list L_2 into list L_1. Then $L_1(1) = 2^{nd}$ Ins $\sqrt{2}$ inserts at the top of list L_1 the value $y_0 = \sqrt{1+1^2} = \sqrt{2}$ and scrolling down to the bottom of list L_1 we can delete $y_n = \sqrt{1+x_n^2}$. The result is that list L_1 is the sequence of values y_{i-1} for $i=1,\dots,n$ so $\text{sum}(L_1) \div n$ produces the L_n estimate.

(iii) As the formula above indicates, we can calculate T_n by $\frac{1}{2}(L_n+R_n)$

(iv) We calculate M_n by first using

$$L_3 = \text{seq}\left(\sqrt{(1 + (1 + I/n - 1/2n)^2)}, I, 1, n \right)$$

and then using $\text{sum}(L_3) \div n$

(v) The results of these calculations to 9 decimal places are given in the table below. For later classroom discussion, the table includes values for the Simpson's Rule estimate $S_{2n} = \frac{2}{3} M_n + \frac{1}{3} T_n$ although the problem instructions didn't ask for this.

n	L_n	R_n	T_n	M_n	S_{2n}
10	1.769155461	1.851340903	1.810248181	1.810014142	1.8100922255
100	1.805984429	1.814202973	1.810093701	1.810091136*	1.81009214*

* not available

The manual for the TI-83 promises reliability to 8 decimal places with the entry in the 9th decimal place unreliable due to round-off errors.

Discussion: Since $f(x) = \sqrt{1+x^2}$ is an increasing function on $[1, 2]$ we know that $L_n \leq I \leq R_n$ for every n.

Hence I is between $L_{100} \approx 1.806$ and $R_{100} \approx 1.814$

We note that T_{10}, M_{10}, and S_{20} are between L_{100} and R_{100} so these must be good estimates of I to 2 decimal places.

As we'll discuss in class, since the graph of f is concave down, $M_n \leq I \leq T_n$ for every n so I lies between $T_{100} \approx 1.810093$ and $M_{100} \approx 1.810091$

We note that T_{10} and M_{10} are outside this range but S_{20} is within it, hence is an accurate estimate of I to 5 decimal places.

(b) The TI-83 command FnInt(√(1+x^2), x, 1, 2)
[FnInt(is obtained by punching MATH 9] yields
1.81009214 as an estimate of I to within 8 decimal places. Note that this number is exactly S_{200}.
The TI-83 uses Simpson's Rule to estimate integrals with a built-in program to stop when successive Simpson's Rule estimates are within 10^{-8} of each other.

Subtracting 1.81009214 from each of our 10 estimates for I in part (a), we obtain the following table of errors with + or — indicating whether the estimate is too big or too small.

n	Error in L_n	Error in R_n	Error in T_n	Error in M_n	Error in S_{2n}
10	−.041	.041	.00016	−.00008	.0000001
100	−.0041	.0041	.0000016	−.0000008	0 to 8 decimal places

Observations: (1) Increasing n by a factor of 10 from 10 to (10)(10) = 100 reduces errors in L_n and R_n by $1/10$ but reduces errors in T_n and M_n by $(1/10)^2$. Theory says errors in S_{2n} are reduced by $(1/10)^4$ so the true error in S_{200} is $\approx 10^{-4}\cdot 10^{-7} = 10^{-11}$.

(2) The errors in M_n are of opposite sign than those of T_n but twice as small in magnitude. This helps explain why $S_{2n} = \dfrac{M_n + M_n + T_n}{3}$ is such a great estimate

(3) S_{200}, S_{20}, M_{100}, T_{100}, M_{10}, T_{10}, R_{100}, L_{100}, R_{10}, L_{10} ranks our 10 estimates in order of increasing error magnitude. S_{20} is 8 times better than M_{100} while T_{10} is around 25 times better than either L_{100} or R_{100}

② [# 22, p.377]

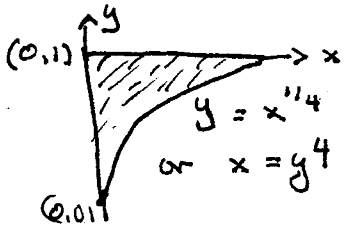
$$\int_1^2 \frac{4+u^2}{u^3}\,du = 4\int_1^2 u^{-3}\,du + \int_1^2 \frac{du}{u}$$

$$= \frac{4\,u^{-2}}{-2}\Big]_1^2 + \ln u\Big]_1^2$$

$$= \left(-\frac{2}{4} + 2\right) + \ln 2 - \cancel{\ln 1}_{0}$$

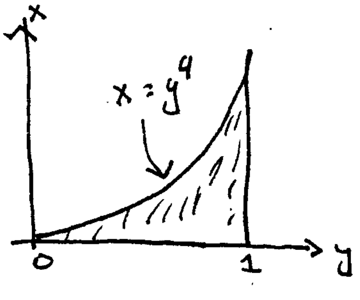
$$= \boxed{3/2 + \ln 2}$$

③ [# 50, p. 378]



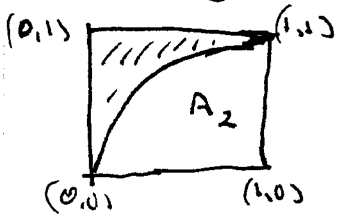
1st Method

The shaded area lies between the y-axis and the curve $x = y^4$ for y from 0 to 1. We can also picture it by reversing the usual roles of the x and y-axes



$$\text{Area} = \int_0^1 y^4\,dy = \frac{y^5}{5}\Big]_0^1 = \boxed{\frac{1}{5}}$$

2nd Method



Shaded Area = area of the square − A_2

$$= 1 - \int_0^1 x^{1/4}\,dx$$

$$= 1 - \left(\frac{4}{5}x^{5/4}\right)\Big]_0^1$$

$$= 1 - 4/5 = \boxed{1/5}$$

(4) $\boxed{\#\ 26,\ pp.\ 387-388}$

Given: V = initial value of computer system

$f(t)$ = depreciation rate at time t (in months)

$g(t)$ = rate of maintenance costs at time t

(a) We define $C(t) = \frac{1}{t}\int_0^t [f(s)+g(s)]\,ds$

The integral gives the total cost of the system over the interval $[0,t]$ due to depreciation and maintenance. Dividing by t gives the average cost per month over the interval $[0,t]$

The company wants to minimize its average monthly cost. The minimum of $C(t)$ must occur at a time t where

$$0 = \frac{dC}{dt} = \frac{d}{dt}\left(\frac{1}{t}\right)\int_0^t [f(s)+g(s)]\,ds + \frac{1}{t}\frac{d}{dt}\int_0^t [f(s)+g(s)]\,ds$$

(by the Fund Thm) $= -\frac{1}{t^2}\int_0^t [f(s)+g(s)]\,ds + \frac{1}{t}\left(f(t)+g(t)\right)$

$$= \frac{1}{t}\left\{-C(t) + (f(t)+g(t))\right\}$$

$$\Longrightarrow \quad C(t) = f(t)+g(t)$$

b) If $f(t) = \begin{cases} \frac{V}{15} - \frac{V}{450}t & \text{for } 0<t\le 30 \\ 0 & \text{for } t>30 \end{cases}$

$D(t)$ = total depreciation over $[0,t] = \int_0^t f(s)\,ds$

$$= \begin{cases} \frac{Vt}{15} - \frac{Vt^2}{900} & \text{for } 0<t\le 30 \\ 2V - V = V & \text{for } t>30 \end{cases}$$

Since $D(t)$ steadily increases for $0<t\le 30$, $\boxed{T=30}$ is the first time at which $D(T) = V$

(c) For $f(t)$ as in (b) and $g(t) = \dfrac{Vt^2}{12\,900}$

$$C(t) = \frac{1}{t}\int_0^t [f(s)+g(s)]\,ds = \frac{1}{t}\left\{D(t) + \frac{Vt^3}{3(12\,900)}\right\}$$

$$= \frac{V}{15} - \frac{V(t)}{900} + \frac{Vt^2}{3(12,900)}$$

$$= \frac{V}{15}\left\{1 - \frac{t}{60} + \frac{t^2}{2580}\right\}$$

for $0 < t \le T = 30$

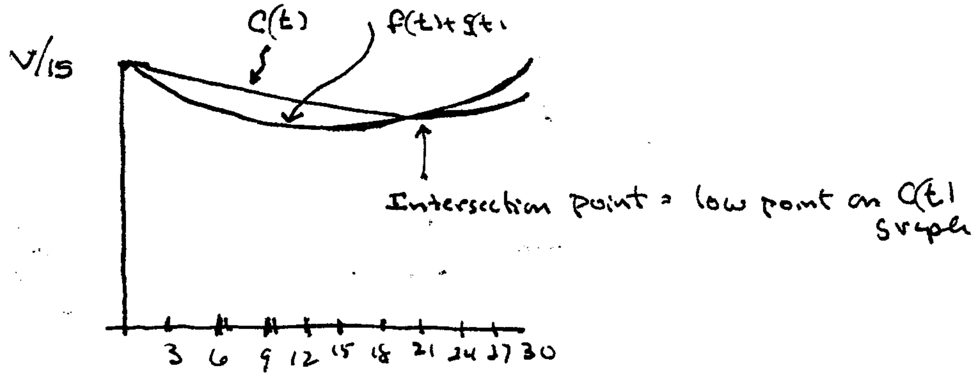
The minimum value occurs when $C'(t) = 0$

i.e. $-\frac{1}{60} + \frac{2t}{2580} = 0$

or $\boxed{t = 21.5 \text{ months}}$

with $C(21.5) =$ minimal cost $= \boxed{.0547\ V}$

(d)



Both $f(t)+g(t) = V/15\,\{1 - t/30 + t^2/860\}$

and $C(t) = V/15\,\{1 - t/60 + \frac{t^2}{2580}\}$

are quadratic functions equal to $V/15$ at $t=0$

The vertex (low point) for $f(t)+g(t)$ occurs roughly at $t=15$ while from (c), the vertex for $C(t)$ occurs at $t = 21.5$

⑤ $\boxed{\#48,\ p.395}$

$$\int_0^4 \frac{x\,dx}{\sqrt{1+2x}} \;=\; \int_{1+2\cdot 0}^{1+2\cdot 4} \frac{1}{4}\left(\frac{u-1}{\sqrt{u}}\right) du$$

$$\boxed{\begin{aligned} u &= 1+2x \\ x &= (u-1)/2 \\ dx &= du/2 \end{aligned}}$$

$$= \frac{1}{4}\int_1^9 \left(u^{1/2} - u^{-1/2}\right) du$$

$$= \frac{1}{4}\left(\frac{2}{3}u^{3/2} - 2u^{1/2}\right)\Big]_1^9$$

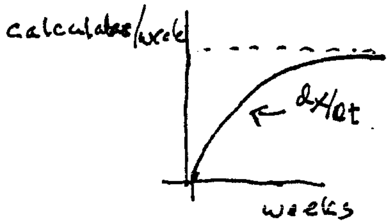
$$= \frac{1}{4}\left(\frac{2}{3}(27-1) - 2(3-1)\right)$$

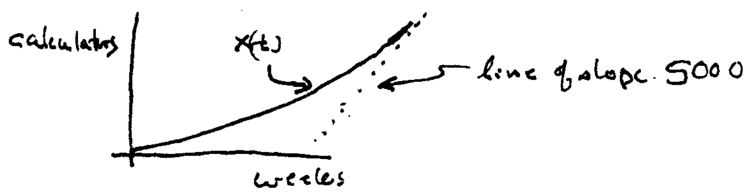
$$= \boxed{10/3}$$

⑥ $\boxed{\#60,\ p.396}$

Given: $x(t)$ = # of calculators produced during the first t weeks

dx/dt = production rate $= 5000\left(1 - \frac{100}{(t+10)^2}\right)$ calculators/w





$x(4) - x(2)$ = # of calculators produced during weeks 3 and 4

$$= \int_2^4 5000\left(1 - \frac{100}{(t+10)^2}\right) dt$$

$$= 5000\left(t + \frac{100}{t+10}\right)\Big]_2^4$$

$$= 5000\left(4-2 + \frac{100}{14} - \frac{100}{12}\right)$$

$$= \boxed{4047.6 \text{ calculators}}$$

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HW # 3 SOLUTIONS

① [# 24, p. 427]

The table of data values gives us 11 velocity values $v(0), v(0.5), v(1), \ldots, v(4.5), v(5)$. We can interpret this __either__ as endpoint data $v(0), v(1), v(2), v(3), v(4), v(5)$ for 5 intervals each of length 1 along with midpoint data $v(0.5), v(1.5), v(2.5), v(3.5), v(4.5)$ for these 5 intervals __or__ as endpoint data for 10 intervals. This allows two ways to compute S_{10}:

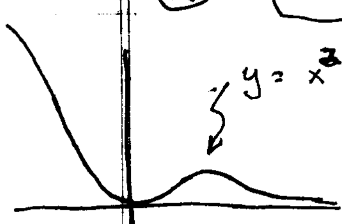
__Method 1__: $S_{10} = \frac{2}{3} M_5 + \frac{1}{3} T_5$

__Method 2__: $S_{10} = \frac{1}{6}\left(\frac{5-0}{5}\right)\{ v(0) + 4v(0.5) + 2v(1) + 4v(1.5) + \cdots + 4v(3.5) + 2v(4) + 4v(4.5) + v(5) \}$

The result by either method is

$$\boxed{S_{10} = 44.735}$$

② $\boxed{\text{\# 14, p. 436}}$



For $x < 0$, $-x^3 > 0$ so $e^{-x^3} \to +\infty$ as $x \to -\infty$. This make us strongly suspect that $\int_{-\infty}^{0} x^2 e^{-x^3} dx$ doesn't converge. Indeed, by guesswork or substitution,

$$\int x^2 e^{-x^3} dx = -\frac{1}{3} e^{-x^3}$$

so $\lim_{s \to -\infty} \int_s^0 x^2 e^{-x^3} dx = \lim_{s \to -\infty} +\frac{1}{3} e^{-s^3} - \frac{1}{3} = \infty$

$\therefore \int_{-\infty}^{\infty} x^2 e^{-x^3} dx = \int_{-\infty}^{0} x^2 e^{-x^3} dx + \int_0^{\infty} x^2 e^{-x^3} dx$ doesn't converge.

__Note__: There's no problem with $\int_0^\infty x^2 e^{-x^3} dx = -\frac{1}{3} e^{-x^3} \Big|_0^\infty = 0 + \frac{1}{3}$

③ [#28, p 436]

Using the substitution $r = 2\tan\Theta$ with

$r^2+4 = 4\sec^2\Theta$, $dr = 2\sec^2 d\Theta$

$\int \frac{dr}{r^2+4} = \frac{1}{2}\int d\Theta = \frac{1}{2}\Theta = \frac{1}{2}\arctan r/2$

As $r \to \infty$, $\arctan r/2 \to \pi/2$

$r \to -\infty$, $\arctan -r/2 \to -\pi/2$

It follows that

$\int_{-\infty}^{\infty} \frac{dr}{r^2+4}$ converges to $\frac{1}{2}\left(\frac{\pi}{2} - (-\pi/2)\right) = \boxed{\pi/2}$

④ [#28, p.436]

The function $\frac{1}{4y-1}$ "blows up" [is undefined] at $y = 1/4$

with $\lim_{t\to 1/4^+} \int_t^1 \frac{dy}{4y-1} = \lim_{t\to 1/4^+} \frac{1}{4}\ln(4y-1)\Big|_t^1 = \infty$

and $\lim_{s\to 1/4^-} \int_0^s \frac{dy}{4y-1} = \lim_{s\to 1/4^-} \frac{1}{4}\ln(4y-1)\Big|_0^s = -\infty$

Since both $\int_{1/4}^{1} \frac{dy}{4y-1}$ and $\int_0^{1/4} \frac{dy}{4y-1}$ diverge,

so does $\int_0^1 \frac{dy}{4y-1}$

⑤ [#42, p.436]

For all $x \geq 1$ $\frac{1}{\sqrt{x^3+1}} \leq \frac{1}{\sqrt{x^3}} = x^{-3/2}$

Since $\int_1^{\infty} x^{-3/2}dx$ converges to $-2x^{-1/2}\Big|_1^{\infty} = 2$

the Comparison Theorem implies that $\int_1^{\infty} \frac{dx}{\sqrt{x^3+1}}$ converges

By the TI-83, $\int_1^{10,000} \frac{dx}{\sqrt{x^3+1}} = 1.875$ so the value of $\int_1^{\infty} \frac{dx}{\sqrt{x^3+1}}$

is just a little bit less than 2.

⑥ [# 50(a) and (b)]

(a) We have $\int_0^\infty e^{-x}\,dx = -e^{-x}\Big|_0^\infty = 0+1 = 1$

Using integration by parts with $u = x^n$, $dv = e^{-x}dx = d(-e^{-x})$ and the fact that $x^n e^{-x}$ vanishes both at 0 and as $x \to \infty$,

$$(*) \quad \int_0^\infty x^n e^{-x}\,dx = -x^n e^{-x}\Big|_0^\infty + n\int_0^\infty x^{n-1} e^{-x}\,dx$$
$$= n\int_0^\infty x^{n-1} e^{-x}\,dx \quad \text{for all } n \geq 1$$

Taking $n=1$ in (*)
$$\int_0^\infty x e^{-x}\,dx = 1\int_0^\infty e^{-x}\,dx = 1 = 1!$$

Then taking $n=2$ in (*)
$$\int_0^\infty x^2 e^{-x}\,dx = 2\int_0^\infty x e^{-x}\,dx = 2\cdot 1 = 2!$$

Next taking $n=3$ in (*)
$$\int_0^\infty x^3 e^{-x}\,dx = 3\int_0^\infty x^2 e^{-x}\,dx = 3\cdot 2\cdot 1 = 3!$$

(b) Continuing to take increasing values of n in (*) and using (a)
$$\int_0^\infty x^4 e^{-x}\,dx = 4\cdot(3!) = 4!$$
$$\int_0^\infty x^5 e^{-x}\,dx = 5\cdot(4!) = 5!$$

And we are led to guess that
$$\int_0^\infty x^n e^{-x}\,dx = n! \quad \text{for each integer } n \geq 1$$

The relation (*) and mathematical induction give a proof of this formula.

⑦ For any $N \geq 1$, $\int_0^N \left[\frac{x}{x^2+1} - \frac{c}{3x+1}\right]dx = \left\{\frac{1}{2}\ln(x^2+1) - \frac{c}{3}\ln(3x+1)\right\}\Big|_0^N$
$$= \frac{1}{2}\ln(N^2+1) - \frac{c}{3}\ln(3N+1) - 0$$

If we take $\boxed{C=3}$, $\frac{1}{2}\ln(N^2+1) - \ln(3N+1)$
$$= \ln\frac{\sqrt{N^2+1}}{3N+1} = \ln\frac{\sqrt{1+1/N^2}}{3+1/N} \to \ln 1/3$$
as $N \to \infty$

So $\int_0^\infty \left[\frac{x}{x^2+1} - \frac{3}{3x+1}\right]dx$ converges to $\ln 1/3 = \boxed{-\ln 3}$

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HM# ~~3~~ SOLUTIONS

① [# 24, p.427]

Method 1: Using the data values given in the table

$$T_5 = \frac{5-0}{5}\left[\frac{v(0)+v(1)}{2} + \frac{v(1)+v(2)}{2} + \frac{v(2)+v(3)}{2} + \frac{v(3)+v(4)}{2} + \frac{v(4)+v(5)}{2}\right]$$

$$= \frac{0+7.34}{2} + \frac{7.34+9.73}{2} + \frac{9.73+10.51}{2} + \frac{10.51+10.76}{2} + \frac{10.76+10.81}{2}$$

$$= 43.745$$

$$M_5 = \frac{5-0}{5}\left[v(0.5) + v(1.5) + v(2.5) + v(3.5) + v(4.5)\right]$$

$$= 4.67 + 8.86 + 10.22 + 10.67 + 10.81$$

$$= 45.23$$

$$S_{10} = \frac{2}{3}M_5 + \frac{1}{3}T_5 = \boxed{44.735}$$

Method 2: $$S_{10} = \frac{1}{3}\left(\frac{5-0}{10}\right)\left[v(0) + 4v(0.5) + 2v(1) + 4v(1.5) + 2v(2) + 4v(2.5) + 2v(3) + 4v(3.5) + 2v(4) + 4v(4.5) + v(5)\right]$$

$$= \boxed{44.735}$$

② [# 14, p. 436]

From the text $\int_{-\infty}^{\infty} e^{-x^2}dx$ converges to $\sqrt{\pi}$

Using integration by parts with $u = x/2$, $dv = 2xe^{-x^2}dx = d(-e^{-x^2})$

$$\int_{-\infty}^{\infty} x^2 e^{-x^2}dx = \underbrace{-x/2\, e^{-x^2}\Big|_{-\infty}^{\infty}}_{0} + \frac{1}{2}\int_{-\infty}^{\infty} e^{-x^2}dx$$

$$= \boxed{\frac{1}{2}\sqrt{\pi}}$$

In particular, the integral converges.