Math 2200 Spring 2017, Exam 3

You may use *any* calculator. You may use ONE "cheat sheet" in the form of a 4" x 6" note card (the medium size of the standard three sizes).

The exam is out of 100 points. All 20 problems are worth 5 points each.

1. In this problem and the one that follows, X is the number of dots that are face-up when a fairly-balanced, 6-sided, *non-standard* die is rolled. The die is nonstandard because five sides have one dot and one side has five dots (and there are no faces with 2, 3, 4, or 6 dots). What is Var(X)?

   A) $\frac{4}{3}$ B) $\frac{13}{9}$ C) $\frac{14}{9}$ D) $\frac{5}{3}$ E) $\frac{16}{9}$
   F) $\frac{17}{9}$ G) 2 H) $\frac{19}{9}$ I) $\frac{20}{9}$ J) $\frac{7}{3}$

Solution. ***Solution*** ***I) 20/9***

The set of values X may assume is {1,5}. If f is the probability function of X, then $f(1) = 5/6$ *and* $f(5) = 1/6$. *Therefore,*

$$E(X) = 1 \times \frac{5}{6} + 5 \times \frac{1}{6} = \frac{5}{3}$$

and

$$Var(X) = E\left(X^2\right) - \left(E\left(X\right)\right)^2 = 1^2 \times \frac{5}{6} + 5^2 \times \frac{1}{6} - \left(\frac{5}{3}\right)^2 = \frac{20}{9}.$$

Alternatively,

$$Var(X) = \left(1 - \frac{5}{3}\right)^2 \left(\frac{5}{6}\right) + \left(5 - \frac{5}{3}\right)^2 \left(\frac{1}{6}\right) = \frac{20}{9}.$$

If you are wary of theoretical calculations, then you can always resort to a simulation as a verification. Note that $20/9 = 2.222\ldots$. *We will roll the die 5000 times and calculate the variance of our simulated rolls. If our theoretical calculation is correct, then our simulation should result in a variance that is in the ballpark of* $20/8 = 2.222\ldots$.

```
> die = c(1,1,1,1,1,5)
> simulated.roll = numeric(5000)
> for(i in 1:5000)
+ {
+ simulated.roll[i] = die[sample(1:6,1)]
+ }
> var(simulated.roll)
[1] 2.238988
```

2. The die of the preceding problem is rolled 4 times. The results of the four rolls are independent. What is the variance of the sample mean?

   A) $\frac{5}{9}$ B) $\frac{2}{3}$ C) $\frac{7}{9}$ D) $\frac{8}{9}$ E) 1
   F) $\frac{10}{9}$ G) $\frac{11}{9}$ H) $\frac{4}{3}$ I) $\frac{13}{9}$ J) $\frac{14}{9}$

Solution. ***Solution*** *A) 5/9*

The variance of X is $\sigma^2 = 20/9$*, the sample size is* $n = 4$*, and so the variance of* $\overline{X}$ *is* $S^2 = \sigma^2/n = 20/(9 \times 4) = 5/9$.

Again, if you are wary of theoretical calculations, then you can resort to a simulation as a verification. Note that $5/9 = 5.555\ldots$. *We will roll the die 20,000 times and calculate the variance of the 5000 sample means of size 4. If our theoretical calculation is correct, then our simulation should result in a variance that is in the ballpark of* $5/9 = 5.555\ldots$.

```
> simulated.sample.mean = numeric(5000)
> for(i in 1:5000)
+ {
+ simulated.sample.mean[i]
+  = (die[sample(1:6,1)] + die[sample(1:6,1)]
+        + die[sample(1:6,1)] + die[sample(1:6,1)])/4
+ }
> var(simulated.sample.mean)
[1] 0.5509892
```

3. Globally, the gender ratio at birth is 1000 boys born for every 934 girls born. Let X be the number of boy babies in a random sample of 1,000,000 global births. What is the standard deviation of X?

   A) 499.1 B) 499.2 C) 499.3 D) 499.4 E) 499.5
   F) 499.6 G) 499.7 H) 499.8 I) 499.9 J) 500.0

Solution. ***Solution*** *G) 499.7*

```
> p = 1000/(1000+934)
> q = 1 - p
> n = 10^6
> sqrt(n*p*q)
[1] 499.7088
```

4. Suppose that X is a binomial random variable with probability 1/3 and size 18. Suppose that Y is a binomial random variable with probability 3/5 and size 75. Suppose also that X and Y are independent. What is the variance of 2X + Y?

   A) 25 B) 26 C) 27 D) 28 E) 29
   F) 30 G) 31 H) 32 I) 33 J) 34

Solution. ***Solution*** *J) 34*

We calculate

$$Var(2X + Y) = 4\ Var(X) + Var(Y) = 4\,(18)\left(\frac{1}{3}\right)\left(\frac{2}{3}\right) + (75)\left(\frac{3}{5}\right)\left(\frac{2}{5}\right) = 34.$$

Once again we can resort to a simulation as a verification.

```
> X.values = c(1,0,0)
> Y.values = c(1,1,1,0,0)
> r.v. = numeric(10000)
> for(i in 1:10000)
+ {
+ r.v.[i] = 2*sum(X.values[sample(1:3,18,replace=TRUE)]) + sum(Y.values[sample(1:5,75,replace=TRUE)])
+ }
> var(r.v.)
[1] 33.83892
```

5. At First President University, the scores on the first midterm of a statistics course were normally distributed with mean 87.10 and standard deviation 13.75. The scores on the second midterm were normally distributed with mean 80.86 and standard deviation 16.13. If two students are randomly selected, what is the probability that the first midterm score of the first student is less than the second midterm score of the second student?

   A) 0.3030 B) 0.3233 C) 0.3436 D) 0.3639 E) 0.3842
   F) 0.4045 G) 0.4248 H) 0.4451 I) 0.4654 J) 0.4857

Solution. ***Solution*** ***E) 0.3842***

Let X be the first midterm score of the first student and let Y be the second midterm score of the second student. Then the mean of $X-Y$ is $87.10-80.86$, or 6.24, and

$$Sd(X-Y) = \sqrt{Var(X)+Var(Y)} = \sqrt{(13.75)^2+(16.13)^2} = 21.19527.$$

Therefore $X-Y \sim N(6.24, 21.19527)$. It follows that

$$\begin{aligned} P(X<Y) &= P(X-Y<0) \\ &= P\left(\frac{X-Y-6.24}{21.19527} < \frac{0-6.24}{21.19527}\right) \\ &= P(Z<-0.2944053) \\ &= \Phi(-0.2944053) \\ &= 1-\Phi(0.2944053) \\ &= 1-0.6157759 \\ &= 0.3842241. \end{aligned}$$

6. Suppose that X_1, X_2, X_3 is a random sample from $N(\mu,\sigma)$. What is the probability that X_1+X_2 exceeds $2X_3$ by σ?

   A) 0.0020 B) 0.0699 C) 0.1378 D) 0.2057 E) 0.2736
   F) 0.3415 G) 0.4094 H) 0.4773 I) 0.5452 J) 0.6131

Solution. ***Solution*** ***F) 0.3415***

First observe that $X_1 + X_2 - 2\,X_3 \sim N\left(\mu + \mu - 2\mu, \sqrt{\sigma^2 + \sigma^2 + 4\,\sigma^2}\right) = N\left(0, \sigma\sqrt{6}\right)$. *It follows that*

$$\begin{aligned} P(X_1 + X_2 > 2\,X_3 + \sigma) &= P(X_1 + X_2 - 2\,X_3 > \sigma) \\ &= P\left(\frac{X_1 + X_2 - 2\,X_3}{\sigma\sqrt{6}} > \frac{\sigma}{\sigma\sqrt{6}}\right) \\ &= P\left(Z > \frac{1}{\sqrt{6}}\right) \\ &= 1 - \Phi(0.4082483) \\ &= 1 - 0.6584543 \\ &= 0.3415457. \end{aligned}$$

7. Suppose that $X_1, X_2, \ldots, X_{17}$ are independent, identically distributed random variables with mean 55 and variance 47. What is the standard deviation of $\overline{X}$?

   A) 1.1914 B) 1.6627 C) 2.1340 D) 2.6053 E) 3.0766
   F) 3.5479 G) 4.0192 H) 4.4905 I) 4.9618 J) 5.4331

Solution. ***Solution*** ***B) 1.6627***

We have

$$Sd\left(\overline{X}\right) = \frac{1}{\sqrt{17}}\,Sd((X_1)) = \frac{\sqrt{47}}{\sqrt{17}} = 1.66274.$$

8. Suppose that the number of goals per game in the National Hockey League in the 2015/2016 season was a Poisson random variable. There were 6012 goals scored in the 1103 NHL games that season. If a random game from that season is selected and the score is looked up, what is the probability that four goals were scored?

   A) 0.0643 B) 0.0955 C) 0.1267 D) 0.1579 E) 0.1891
   F) 0.2203 G) 0.2515 H) 0.2827 I) 0.3139 J) 0.3451

Solution. ***Solution*** ***D) 0.1579***

The mean number λ *of goals per game is given by* $\lambda = 6012/1103 = 5.450589$. *Therefore,*

$$P(\textit{four goals were scored}) = \exp(-\lambda)\,\frac{\lambda^4}{4!} = 0.1579071.$$

Here is the solution using R:

```
> lambda = 6012/1103
> dpois(4,lambda)
[1] 0.1579071
```

9. For a particular blood sample, the number of bacteria in a square of a hemocytometer is a Poisson random variable with mean equal to 2.03125. What is the probability that a square has more than 2 bacteria?

A) 0.3318 B) 0.3529 C) 0.3740 D) 0.3951 E) 0.4162
F) 0.4373 G) 0.4584 H) 0.4795 I) 0.5006 J) 0.5217

Solution. ***Solution*** ***A) 0.3318***

Let $\lambda = 2.03125$. *Then the requested probability is*

$$1 - e^{-\lambda}\left(1 + \lambda + \frac{\lambda^2}{2}\right) = 1 - (0.1311715)(1 + 2.03125 + 2.062988) = 1 - 0.6682188 = 0.3317812.$$

Here are three ways to get the solution using R:

```
> ppois(2,2.03125,lower.tail=FALSE)
[1] 0.3317814
> 1 - ppois(2,2.03125)
[1] 0.3317814
> 1 - sum(dpois(0:2,2.03125))
[1] 0.3317814
```

10. In the 2016/2017 academic year, exactly 7543 undergraduates are enrolled at First President University. A totally unrelated fact is that the number of babies born per minute in the world is a random variable with mean 255 and standard deviation 16. (That is actually a UNICEF best estimate that we will take as a fact.) Assume that, for any two non-overlapping minutes, the numbers of babies born are independent and identically distributed. What is the probability that more than 7657 babies were born during the first half hour of this exam. Use the normal approximation with correction for continuity.

A) 0.4254 B) 0.4335 C) 0.4416 D) 0.4497 E) 0.4578
F) 0.4659 G) 0.4740 H) 0.4821 I) 0.4902 J) 0.4983

Solution. ***Solution*** ***F) 0.4659***

Let X_j *be the number of babies born in the j'th minute. Notice that the mean of* $X_1 + X_2 + \cdots + X_{30}$ *is* 30×255, *or 7650, and the standard deviation is* $\sqrt{30} \times 16$, *or 87.63561. Let* Y *be a normal random variable with mean 7650 and standard deviation 87.63561. Then*

$$\begin{aligned}
P(X_1 + X_2 + \cdots + X_{30} > 7657) &= P(X_1 + X_2 + \cdots + X_{30} \geq 7658) \\
&\approx P(Y \geq 7657.5) \\
&= P\left(\frac{Y - 7650}{87.63561} \geq \frac{7657.5 - 7650}{87.63561}\right) \\
&= P(Z \geq 0.08558165) \\
&= 1 - \Phi(0.08558165) \\
&= 0.4658995.
\end{aligned}$$

11. This year in the NCAA men's basketball championship, the team from Gonzaga University has made it to the Final Four. A guard on that team, Nigel Williams-Goss, has a free throw percentage of 89.5. Use the normal approximation to estimate the probability that he will make exactly 17 baskets in 20 free throws. (You might wonder if it is advisable to use the normal approximation, but it is because you have been told to do so.)

A) 0.1638	B) 0.1751	C) 0.1864	D) 0.1977	E) 0.2090
F) 0.2203	G) 0.2316	H) 0.2429	I) 0.2542	J) 0.2655

Solution. ***Solution*** *G) 0.2316*

Here $p = 0.895$, $q = 0.105$, *and* $n = 20$. *The sample size 20 is generally too small for a normal approximation. That is particularly true when a binomial distribution with p close to 1 is being approximated. But the instructions are to go ahead. We will do so, but we will not expect that our approximation is very accurate. Let* $X_j = 1$ *if the j'th free throw is successful and* $X_j = 0$ *if not. We need to approximate* $X_1 + X_2 + \cdots + X_{20}$, *which has mean* 20×0.895, *or 17.9, and the standard deviation is* $\sqrt{20 \times 0.895 \times 0.105}$, *or 1.370949. Let Y be a normal random variable with mean 17.9 and standard deviation 1.370949. Then*

$$\begin{aligned}
P(X_1 + X_2 + \cdots + X_{20} = 17) &\approx P(16.5 < Y < 17.5) \\
&= P\left(\frac{16.5 - 17.9}{1.370949} < \frac{Y - 17.9}{1.370949} < \frac{17.5 - 17.9}{1.370949}\right) \\
&= P(-1.02119 < Z < -0.2917687) \\
&= \Phi(-0.2917687) - \Phi(-1.02119) \\
&= \Phi(1.02119) - \Phi(0.2917687) \\
&= 0.8464178 - 0.6147683 \\
&= 0.2316495.
\end{aligned}$$

As expected, the approximation is not very accurate. In the next grab from an R session, the normal approximation is calculated in the first three input lines followed by a one-line calculation of the actual value. As you can see, the error resulting from the normal approximation is about 15%.

```
> m = 20* 0.895
> s = sqrt(20*0.895*0.105)
> pnorm(17.5,mean=m,sd=s) - pnorm(16.5,mean=m,sd=s) # normal approximation
[1] 0.2316497
> dbinom(17, size = 20, prob = 0.895) # actual value
[1] 0.2002001
```

12. A manufacturer of speedboats intends to make a large purchase of motors from a supplier. The motors are rated at 225 HP (horsepower), but, in fact, the horsepower of the motors is a normal random variable with mean 225 and standard deviation 12.5. The speedboat manufacturer tests a random sample of size 10 with the intent of finding a different supplier if the sample average is below 220 HP. What is the probability of that?

A) 0.0284	B) 0.0657	C) 0.1030	D) 0.1403	E) 0.1776
F) 0.2149	G) 0.2522	H) 0.2895	I) 0.3268	J) 0.3641

Solution. ***Solution*** ***C) 0.1030***

Let $\sigma = 128.5$ *and* $n = 10$. *Then* $\sigma/\sqrt{n} = 3.952847$. *The sample average* $\overline{\mathrm{X}}$ *has distribution N(225,3.952847). We calculate*

$$P\left(\overline{X} < 220\right) = P\left(\frac{\overline{X} - 225}{3.952847} < \frac{220 - 225}{3.952847}\right) = P\left(Z < -1.264911\right) = 0.1029516.$$

13. The Stanford-Binet Intelligence Quotient (IQ) is designed so that scores are normally distributed with mean 100 and standard deviation 16. Let S^2 be the sample variance of the measured IQs of a random sample of size 17. For what value of v is P $\left(S^2 < v\right) = 0.800$?

A) 310.454	B) 312.578	C) 314.701	D) 316.824	E) 318.948
F) 321.071	G) 323.195	H) 325.318	I) 327.441	J) 329.565

Solution. ***Solution*** ***I) 327.441***

Because $(17-1) \times S^2/16^2$ *has* χ^2_{17-1} *for its distribution, we have*

$$P\left(S^2 < v\right) = P\left(\frac{(17-1)\,S^2}{16^2} < \frac{v}{16}\right) = P\left(\chi^2_{16} < \frac{v}{16}\right) = 0.800.$$

Therefore, $P\left(\chi^2_{16} > \frac{v}{16}\right) = 1 - 0.800 = 0.200$. *From the given table, we see that* $v/16 = 20.4651$, *or* $v = 16 \times 20.4651$, *or* $v = 327.4416$.

14. Suppose that the number of hours of sleep of a First President University student is normally distributed with unknown mean μ and unknown standard deviation σ. A random sample of 21 students yields a sample mean equal to $\overline{\mathrm{X}} = 6.21$ hours and a sample standard deviation equal to $s = 1.4254$ hours. Calculate P($\mu > 6.25$).

A) 0.01	B) 0.05	C) 0.10	D) 0.15	E) 0.20
F) 0.25	G) 0.30	H) 0.35	I) 0.40	J) 0.45

Solution. ***Solution*** ***J) 0.45***

$$\begin{aligned}
P\left(\mu > 6.25\right) &= P\left(-\mu < -6.25\right) \\
&= P\left(\frac{\overline{X} - \mu}{1.4254/\sqrt{21}} < \frac{6.21 - 6.25}{1.4254/\sqrt{21}}\right) \\
&= P\left(t_{21-1} < -0.1285976\right) \\
&= P\left(t_{20} > 0.1285976\right) \\
&= 0.4494802.
\end{aligned}$$

15. What proportion p of Americans believe that in sporting events, God helps one team win? In 2013, two weeks before the Super Bowl, the Public Religion Research Institute conducted a random telephone survey of size 1033. Asked to agree or disagree with the statement, God plays a role in determining which team wins a sporting event, 279 of the surveyees agreed. Assuming that the poll was well-conducted, what is the upper endpoint of a 95% confidence interval for p?

A) 0.2830	B) 0.2901	C) 0.2972	D) 0.3043	E) 0.3114
F) 0.3185	G) 0.3256	H) 0.3327	I) 0.3398	J) 0.3469

Solution. ***Solution*** ***C) 0.2972***

The requested upper bound is

$$\frac{279}{1033} + z_{0.05/2}\sqrt{\frac{(279)(1033-279)}{1033^3}} = 0.2700871 + 1.959964 \cdot 0.01381457 = 0.2971632.$$

16. In a recent (2013) study conducted at the Kennedy Krieger Institute, a Johns Hopkins affiliate, psychologists and educators investigated whether the use of tablet computers increased task completion among children with autism spectrum disorders. As is frequently the case with such one-on-one studies, the sample size was small: 7 in this study. The results concerning the researchers' primary focus were mixed, but 5 of 7 of the subjects showed increased scores on a common learning assessment exam (LAP-3). Let p be the true proportion of children with ASD who improve their LAP-3 scores after tablet "intervention". What is the lower limit of a 95% confidence interval for p?

A) 0.1389	B) 0.1802	C) 0.2215	D) 0.2628	E) 0.3041
F) 0.3454	G) 0.3867	H) 0.4280	I) 0.4693	J) 0.5106

Solution. ***Solution*** ***D) 0.2628***

The 10-10 Success-Failure Condition is not satisfied. Therefore, the Agresti-Coull adjustment is needed. With the four phony trials included, we have $n' = 7 + 4 = 11$, $\widehat{p}' = (5+2)/11 = 0.6363636$, $\widehat{q}' = 1 - \widehat{p}' = (2+2)/11 = 0.3636364$, $SE(\widehat{p}') = \sqrt{(7)(4)/11^3} = 0.1450407$, $ME(\widehat{p}') = z_{0.025} \times SE(\widehat{p}') = (1.959964)(0.1450407) = 0.2842746$, *and* $\widehat{p}' \pm ME(\widehat{p}')$ *is the interval* $[0.6363636 - 0.2842746, 0.6363636 + 0.2842746]$, *or* $[0.35208900.9206382]$.

17. Hereditary angioedema is a disabling, potentially fatal condition caused by a deficiency of a certain inhibitor protein. A trial preventive therapy involving bi-weekly subcutaneous injections of an inhibitor (CSL830) was reported in the March 2017 issue of the New England Journal of Medicine. To establish a baseline for the number X of angioedema attacks per patient per month, a history of attacks over the three months prior to the trial was recorded. In all, the 45 patients in the study suffered 1458 attacks during the period. The lower endpoint of a 95% c.i. for the mean μ of X was 4.1. What was the sample standard deviation of X?

A) 22.0858	B) 22.2981	C) 22.5104	D) 22.7227	E) 22.9350
F) 23.1473	G) 23.3596	H) 23.5719	I) 23.7842	J) 23.9965

Solution. ***Solution*** ***E) 22.9350***

First note that $n = 45$ *is large enough for a normal approximation. Also note that* X *is not said to be normal, so a student-t distribution is not appropriate. We are to find* S*, the sample standard deviation. The sample mean was* $1458/(3 \times 45)$*, or* 10.8*. The margin of error ME was* $10.8 - 4.1$*, or* $ME = 6.7$*. Thus* $1.959964 \times S/\sqrt{45} = 6.7$*, or* $S = 22.93153$*.*

18. An earlier problem referred to a poll conducted by the Public Religion Research Institute. Of interest was the proportion of Americans that believe God influences sporting events. Suppose that the Institute wanted the width (or length, if you prefer) of a 90% confidence interval for the proportion to be no greater than 0.06. Of the integers below, answer with the *smallest* value for the sample size that meets the confidence level and margin of error requirements.

    A) 374 B) 428 C) 482 D) 536 E) 590
    F) 644 G) 698 H) 752 I) 806 J) 860

Solution. ***Solution*** ***H) 752***

Here $z_{0.05} = 1.644854$ *and* $2\,ME_0 = 0.06$*. (The problem refers to the width of the c.i., not the margin of error, which is the half-width.) The required sample size is* $n = \lceil (1.644854/0.06)^2 \rceil = \lceil 751.5402 \rceil = 752$ *rounded up to the nest whole number.*

19. In the Framingham Heart study, the sample mean of the systolic blood pressure of the 3,534 participants was 127.3. In a small subsample of size 10, the sample mean of the systolic blood pressure was 121.2 with standard deviation 11.1. If the subsample data is used to arrive at a 95% confidence interval for mean systolic blood pressure, what is the upper endpoint? (Assume that systolic blood pressure is normally distributed.)

    A) 127.8501 B) 128.0114 C) 128.1727 D) 128.3340 E) 128.4953
    F) 128.6566 G) 128.8179 H) 128.9792 I) 129.1405 J) 129.3018

Solution. ***Solution*** ***I) 129.1405***

The upper end point is

$$121.2 + t_{0.025,9}\,\frac{11.1}{\sqrt{10}}$$

In R, `121.2 + qt(0.025,df=9,lower.tail=FALSE) * 11.1/sqrt(10)` *gets the required value.*

20. The reference range for hemoglobin in men is 13.5 to 17.5 grams per deciliter. To determine the true mean, which might not be the center of the reference range, what is the smallest sample size for which a 99% confidence interval will be no wider than 0.15 grams per deciliter?

    A) 905 B) 960 C) 1015 D) 1070 E) 1125
    F) 1180 G) 1235 H) 1290 I) 1345 J) 1400

Solution. ***Solution*** ***F) 1180***

Here $z_{0.005} = 2.5758294, ME_0 = 0.15/2 = 0.075$*, and* $S_{RR} = (17.5 - 13.5)/4 = 1$*. The required sample size is* $n = \lceil ((2.5758294)(1)/0.075)^2 \rceil = \lceil 1179.537 \rceil = 1180$ *rounded up to the nest whole number.*

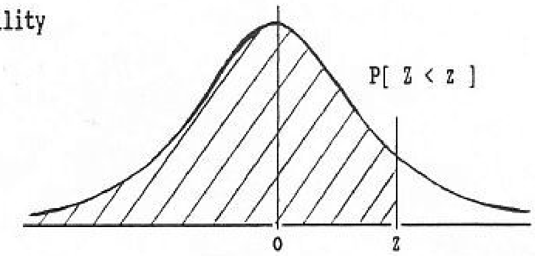
STANDARD STATISTICAL TABLES

1. Areas under the Normal Distribution

The table gives the cumulative probability up to the standardised normal value z

i.e.

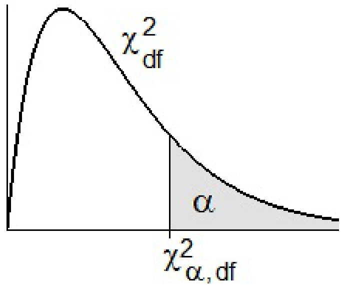
$$P[\,Z < z\,] = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp(-\tfrac{1}{2}Z^2)\, dZ$$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7020	0.7054	0.7089	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8804	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9865	0.9868	0.9871	0.9874	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9924	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9980	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986

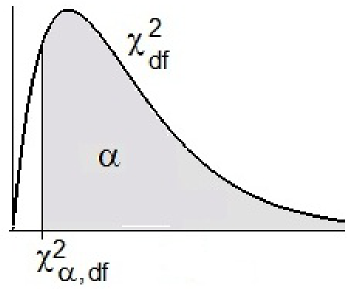
z	3.00	3.10	3.20	3.30	3.40	3.50	3.60	3.70	3.80	3.90
P	0.9986	0.9990	0.9993	0.9995	0.9997	0.9998	0.9998	0.9999	0.9999	1.0000

Values of $\chi^2_{\alpha,df}$ $P(\chi^2_{df} \geq \chi^2_{\alpha,df}) = \alpha$



df \ α	0.005	0.010	0.025	0.050	0.100	0.200	0.250	0.300	0.400	0.500
1	7.8794	6.6349	5.0239	3.8415	2.7055	1.6424	1.3233	1.0742	0.7083	0.4549
2	10.5966	9.2103	7.3778	5.9915	4.6052	3.2189	2.7726	2.4079	1.8326	1.3863
3	12.8382	11.3449	9.3484	7.8147	6.2514	4.6416	4.1083	3.6649	2.9462	2.3660
4	14.8603	13.2767	11.1433	9.4877	7.7794	5.9886	5.3853	4.8784	4.0446	3.3567
5	16.7496	15.0863	12.8325	11.0705	9.2364	7.2893	6.6257	6.0644	5.1319	4.3515
6	18.5476	16.8119	14.4494	12.5916	10.6446	8.5581	7.8408	7.2311	6.2108	5.3481
7	20.2777	18.4753	16.0128	14.0671	12.0170	9.8032	9.0371	8.3834	7.2832	6.3458
8	21.9550	20.0902	17.5345	15.5073	13.3616	11.0301	10.2189	9.5245	8.3505	7.3441
9	23.5894	21.6660	19.0228	16.9190	14.6837	12.2421	11.3888	10.6564	9.4136	8.3428
10	25.1882	23.2093	20.4832	18.3070	15.9872	13.4420	12.5489	11.7807	10.4732	9.3418
11	26.7568	24.7250	21.9200	19.6751	17.2750	14.6314	13.7007	12.8987	11.5298	10.3410
12	28.2995	26.2170	23.3367	21.0261	18.5493	15.8120	14.8454	14.0111	12.5838	11.3403
13	29.8195	27.6882	24.7356	22.3620	19.8119	16.9848	15.9839	15.1187	13.6356	12.3398
14	31.3193	29.1412	26.1189	23.6848	21.0641	18.1508	17.1169	16.2221	14.6853	13.3393
15	32.8013	30.5779	27.4884	24.9958	22.3071	19.3107	18.2451	17.3217	15.7332	14.3389
16	34.2672	31.9999	28.8454	26.2962	23.5418	20.4651	19.3689	18.4179	16.7795	15.3385
17	35.7185	33.4087	30.1910	27.5871	24.7690	21.6146	20.4887	19.5110	17.8244	16.3382
18	37.1565	34.8053	31.5264	28.8693	25.9894	22.7595	21.6049	20.6014	18.8679	17.3379
19	38.5823	36.1909	32.8523	30.1435	27.2036	23.9004	22.7178	21.6891	19.9102	18.3377
20	39.9968	37.5662	34.1696	31.4104	28.4120	25.0375	23.8277	22.7745	20.9514	19.3374
21	41.4011	38.9322	35.4789	32.6706	29.6151	26.1711	24.9348	23.8578	21.9915	20.3372
22	42.7957	40.2894	36.7807	33.9244	30.8133	27.3015	26.0393	24.9390	23.0307	21.3370
23	44.1813	41.6384	38.0756	35.1725	32.0069	28.4288	27.1413	26.0184	24.0689	22.3369
24	45.5585	42.9798	39.3641	36.4150	33.1962	29.5533	28.2412	27.0960	25.1063	23.3367
25	46.9279	44.3141	40.6465	37.6525	34.3816	30.6752	29.3389	28.1719	26.1430	24.3366
30	53.6720	50.8922	46.9792	43.7730	40.2560	36.2502	34.7997	33.5302	31.3159	29.3360
40	66.7660	63.6907	59.3417	55.7585	51.8051	47.2685	45.6160	44.1649	41.6222	39.3353
50	79.4900	76.1539	71.4202	67.5048	63.1671	58.1638	56.3336	54.7228	51.8916	49.3349
60	91.9517	88.3794	83.2977	79.0819	74.3970	68.9721	66.9815	65.2265	62.1348	59.3347
70	104.2149	100.4252	95.0232	90.5312	85.5270	79.7146	77.5767	75.6893	72.3583	69.3345
80	116.3211	112.3288	106.6286	101.8795	96.5782	90.4053	88.1303	86.1197	82.5663	79.3343
90	128.2989	124.1163	118.1359	113.1453	107.5650	101.0537	98.6499	96.5238	92.7614	89.3342
100	140.1695	135.8067	129.5612	124.3421	118.4980	111.6667	109.1412	106.9058	102.9459	99.3341

Chi-Squared Values—Right Tails.



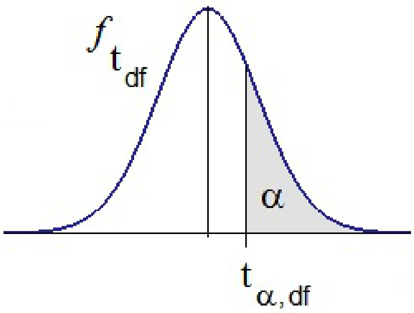
Values of $\chi^2_{\alpha, df}$ $\quad P(\chi^2_{df} \geqslant \chi^2_{\alpha, df}) = \alpha$

df \ α	0.600	0.700	0.750	0.800	0.900	0.950	0.975	0.990	0.995
1	0.2750	0.1485	0.1015	0.0642	0.0158	0.0039	0.0010	0.0002	0.0000
2	1.0217	0.7133	0.5754	0.4463	0.2107	0.1026	0.0506	0.0201	0.0100
3	1.8692	1.4237	1.2125	1.0052	0.5844	0.3518	0.2158	0.1148	0.0717
4	2.7528	2.1947	1.9226	1.6488	1.0636	0.7107	0.4844	0.2971	0.2070
5	3.6555	2.9999	2.6746	2.3425	1.6103	1.1455	0.8312	0.5543	0.4117
6	4.5702	3.8276	3.4546	3.0701	2.2041	1.6354	1.2373	0.8721	0.6757
7	5.4932	4.6713	4.2549	3.8223	2.8331	2.1673	1.6899	1.2390	0.9893
8	6.4226	5.5274	5.0706	4.5936	3.4895	2.7326	2.1797	1.6465	1.3444
9	7.3570	6.3933	5.8988	5.3801	4.1682	3.3251	2.7004	2.0879	1.7349
10	8.2955	7.2672	6.7372	6.1791	4.8652	3.9403	3.2470	2.5582	2.1559
11	9.2373	8.1479	7.5841	6.9887	5.5778	4.5748	3.8157	3.0535	2.6032
12	10.1820	9.0343	8.4384	7.8073	6.3038	5.2260	4.4038	3.5706	3.0738
13	11.1291	9.9257	9.2991	8.6339	7.0415	5.8919	5.0088	4.1069	3.5650
14	12.0785	10.8215	10.1653	9.4673	7.7895	6.5706	5.6287	4.6604	4.0747
15	13.0297	11.7212	11.0365	10.3070	8.5468	7.2609	6.2621	5.2293	4.6009
16	13.9827	12.6243	11.9122	11.1521	9.3122	7.9616	6.9077	5.8122	5.1422
17	14.9373	13.5307	12.7919	12.0023	10.0852	8.6718	7.5642	6.4078	5.6972
18	15.8932	14.4399	13.6753	12.8570	10.8649	9.3905	8.2307	7.0149	6.2648
19	16.8504	15.3517	14.5620	13.7158	11.6509	10.1170	8.9065	7.6327	6.8440
20	17.8088	16.2659	15.4518	14.5784	12.4426	10.8508	9.5908	8.2604	7.4338
21	18.7683	17.1823	16.3444	15.4446	13.2396	11.5913	10.2829	8.8972	8.0337
22	19.7288	18.1007	17.2396	16.3140	14.0415	12.3380	10.9823	9.5425	8.6427
23	20.6902	19.0211	18.1373	17.1865	14.8480	13.0905	11.6886	10.1957	9.2604
24	21.6525	19.9432	19.0373	18.0618	15.6587	13.8484	12.4012	10.8564	9.8862
25	22.6156	20.8670	19.9393	18.9398	16.4734	14.6114	13.1197	11.5240	10.5197
30	27.4416	25.5078	24.4776	23.3641	20.5992	18.4927	16.7908	14.9535	13.7867
40	37.1340	34.8719	33.6603	32.3450	29.0505	26.5093	24.4330	22.1643	20.7065
50	46.8638	44.3133	42.9421	41.4492	37.6886	34.7643	32.3574	29.7067	27.9907
60	56.6200	53.8091	52.2938	50.6406	46.4589	43.1880	40.4817	37.4849	35.5345
70	66.3961	63.3460	61.6983	59.8978	55.3289	51.7393	48.7576	45.4417	43.2752
80	76.1879	72.9153	71.1445	69.2069	64.2778	60.3915	57.1532	53.5401	51.1719
90	85.9925	82.5111	80.6247	78.5584	73.2911	69.1260	65.6466	61.7541	59.1963
100	95.8078	92.1289	90.1332	87.9453	82.3581	77.9295	74.2219	70.0649	67.3276

Chi-Squared Values—Central Hump + Right Tails.

Values of $t_{\alpha, df}$

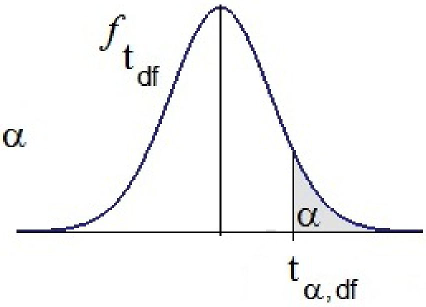
$P(\ t_{df} \geqslant t_{\alpha, df}\ ) = \alpha$



df \ α	.450	.400	.350	.300	.250	.200
1	.1584	.3249	.5095	.7265	1.0000	1.3764
2	.1421	.2887	.4447	.6172	.8165	1.0607
3	.1366	.2767	.4242	.5844	.7649	.9785
4	.1338	.2707	.4142	.5686	.7407	.9410
5	.1322	.2672	.4082	.5594	.7267	.9195
6	.1311	.2648	.4043	.5534	.7176	.9057
7	.1303	.2632	.4015	.5491	.7111	.8960
8	.1297	.2619	.3995	.5459	.7064	.8889
9	.1293	.2610	.3979	.5435	.7027	.8834
10	.1289	.2602	.3966	.5415	.6998	.8791
11	.1286	.2596	.3956	.5399	.6974	.8755
12	.1283	.2590	.3947	.5386	.6955	.8726
13	.1281	.2586	.3940	.5375	.6938	.8702
14	.1280	.2582	.3933	.5366	.6924	.8681
15	.1278	.2579	.3928	.5357	.6912	.8662
16	.1277	.2576	.3923	.5350	.6901	.8647
17	.1276	.2573	.3919	.5344	.6892	.8633
18	.1274	.2571	.3915	.5338	.6884	.8620
19	.1274	.2569	.3912	.5333	.6876	.8610
20	.1273	.2567	.3909	.5329	.6870	.8600
21	.1272	.2566	.3906	.5325	.6864	.8591
22	.1271	.2564	.3904	.5321	.6858	.8583
23	.1271	.2563	.3902	.5317	.6853	.8575
24	.1270	.2562	.3900	.5314	.6848	.8569
25	.1269	.2561	.3898	.5312	.6844	.8562
26	.1269	.2560	.3896	.5309	.6840	.8557
27	.1268	.2559	.3894	.5306	.6837	.8551
28	.1268	.2558	.3893	.5304	.6834	.8546
29	.1268	.2557	.3892	.5302	.6830	.8542
30	.1267	.2556	.3890	.5300	.6828	.8538
40	.1265	.2550	.3881	.5286	.6807	.8507
50	.1263	.2547	.3875	.5278	.6794	.8489
60	.1262	.2545	.3872	.5272	.6786	.8477
70	.1261	.2543	.3869	.5268	.6780	.8468
80	.1261	.2542	.3867	.5265	.6776	.8461
90	.1260	.2541	.3866	.5263	.6772	.8456
100	.1260	.2540	.3864	.5261	.6770	.8452

Values of $t_{\alpha, df}$

$P(t_{df} \geq t_{\alpha, df}) = \alpha$



df \ α	.150	.100	.050	.025	.010	.005
1	1.9626	3.0777	6.3138	12.7062	31.8205	63.6567
2	1.3862	1.8856	2.9200	4.3027	6.9646	9.9248
3	1.2498	1.6377	2.3534	3.1824	4.5407	5.8409
4	1.1896	1.5332	2.1318	2.7764	3.7469	4.6041
5	1.1558	1.4759	2.0150	2.5706	3.3649	4.0321
6	1.1342	1.4398	1.9432	2.4469	3.1427	3.7074
7	1.1192	1.4149	1.8946	2.3646	2.9980	3.4995
8	1.1081	1.3968	1.8595	2.3060	2.8965	3.3554
9	1.0997	1.3830	1.8331	2.2622	2.8214	3.2498
10	1.0931	1.3722	1.8125	2.2281	2.7638	3.1693
11	1.0877	1.3634	1.7959	2.2010	2.7181	3.1058
12	1.0832	1.3562	1.7823	2.1788	2.6810	3.0545
13	1.0795	1.3502	1.7709	2.1604	2.6503	3.0123
14	1.0763	1.3450	1.7613	2.1448	2.6245	2.9768
15	1.0735	1.3406	1.7531	2.1314	2.6025	2.9467
16	1.0711	1.3368	1.7459	2.1199	2.5835	2.9208
17	1.0690	1.3334	1.7396	2.1098	2.5669	2.8982
18	1.0672	1.3304	1.7341	2.1009	2.5524	2.8784
19	1.0655	1.3277	1.7291	2.0930	2.5395	2.8609
20	1.0640	1.3253	1.7247	2.0860	2.5280	2.8453
21	1.0627	1.3232	1.7207	2.0796	2.5176	2.8314
22	1.0614	1.3212	1.7171	2.0739	2.5083	2.8188
23	1.0603	1.3195	1.7139	2.0687	2.4999	2.8073
24	1.0593	1.3178	1.7109	2.0639	2.4922	2.7969
25	1.0584	1.3163	1.7081	2.0595	2.4851	2.7874
26	1.0575	1.3150	1.7056	2.0555	2.4786	2.7787
27	1.0567	1.3137	1.7033	2.0518	2.4727	2.7707
28	1.0560	1.3125	1.7011	2.0484	2.4671	2.7633
29	1.0553	1.3114	1.6991	2.0452	2.4620	2.7564
30	1.0547	1.3104	1.6973	2.0423	2.4573	2.7500
40	1.0500	1.3031	1.6839	2.0211	2.4233	2.7045
50	1.0473	1.2987	1.6759	2.0086	2.4033	2.6778
60	1.0455	1.2958	1.6706	2.0003	2.3901	2.6603
70	1.0442	1.2938	1.6669	1.9944	2.3808	2.6479
80	1.0432	1.2922	1.6641	1.9901	2.3739	2.6387
90	1.0424	1.2910	1.6620	1.9867	2.3685	2.6316
100	1.0418	1.2901	1.6602	1.9840	2.3642	2.6259