Math 2200 Spring 2016, Exam 3

You may use *any* calculator. You may use a 4×6 inch notecard as a cheat sheet.

1. Let X be the distribution consisting of the four numbers 6, 7, 8, 15. Calculate the standard deviation s_{X} of X. Randomly select one of the numbers in the distribution X in such a way that the four possible selections are equally likely. Let Y be the number selected. Calculate the standard deviation s_{Y} of the random variable Y. What is $s_{\mathrm{X}} - s_{\mathrm{Y}}$?

A) -0.5470 B) -0.4102 C) -0.2735 D) -0.1368 E) 0.0000
F) 0.1367 G) 0.2735 H) 0.4102 I) 0.5469 J) 0.6837

Solution. *Answer:* ***I***
We calculate $\mu_X = 9$ *and*

$$s_X = \sqrt{\frac{1}{3}\left((6-9)^2 + (7-9)^2 + (8-9)^2 + (15-9)^2\right)} = 4.082483.$$

Next,

$$E(Y) = 6\,P(Y=6) + 7\,P(Y=7) + 8\,P(Y=8) + 15\,P(Y=15) = \frac{1}{4}\big(6+7+8+15\big) = 9.$$

Therefore

$$\begin{aligned} s_Y &= \sqrt{(6-9)^2\,P(Y=6 + (7-9)^2\,P(Y=7 + (8-9)^2\,P(Y=8 + (15-9)^2\,P(Y=15)} \\ &= \sqrt{\frac{1}{4}\left((6-9)^2 + (7-9)^2 + (8-9)^2 \ + (15-9)^2\right)} \\ &= 3.535534. \end{aligned}$$

The answer is 4.082483 - 3.535534, or 0.546949.

2. Three persons are exposed to a disease, but the chance of contracting the disease from the exposure is only 0.04. The diagnostic strategy is to pool samples of their blood and test it for a marker of the disease. If the pooled blood sample tests negative, then all three persons are free of the disease and no further testing is done. However, if the pooled sample tests positive, then each of the persons is to be tested individually. Let X be the number of tests that will be performed to implement this diagnostic procedure. Calculate the expectation of X. (The next problem will also concern this random variable.)

A) 1.1584 B) 1.2521 C) 1.3458 D) 1.4395 E) 1.5332
F) 1.6269 G) 1.7206 H) 1.8143 I) 1.9080 J) 2.0017

Solution. *Answer:* ***C***
The probability that an exposed person does not contract the disease is 0.96. The probability that all three persons do not contract the disease is $(0.96)^3$. *The probability that at lease one of the three exposed persons contracts the disease is* $1-(0.96)^3$. *The random variable X can assume two values: 1 and 4. From our calculations, we have* $P(X=1) = (0.96)^3$ *and* $P(X=4) = 1-(0.96)^3$. *Thus*

$$E(X) = 1 \times P(X=1) + 4 \times P(X=4) = (0.96)^3 + 4 \times \big(1-(0.96)^3\big) = 1.345792.$$

3. What is the variance of the random variable X of the preceding problem?

A) 0.4613 B) 0.5526 C) 0.6439 D) 0.7352 E) 0.8265
F) 0.9178 G) 1.0091 H) 1.1004 I) 1.1917 J) 1.2830

Solution. *Answer:* ***F***

Using the result E(X) = 1.345792 of the proceding problem, we have

$$\begin{aligned} Var(X) &= (1-1.345792)^2 \times P(X=1) + (4-1.345792)^2 \times P(X=4) \\ &= (1-1.345792)^2 \times (0.96)^3 + (4-1.345792)^2 \times \left(1-(0.96)^3\right) \\ &= 0.9178039. \end{aligned}$$

4. The annual cost of owning a dog is a normal random variable with mean \$695 and standard deviation \$45. The annual cost of owning a cat is a normal random variable with mean \$705 and standard deviation \$35. What is the probability that the total annual cost of owning one dog and two cats exceeds \$2000? (Data from the ASPCA, 2014. Pet health insurance is assumed. One time costs, usually incurred in the first year of ownership, are excluded.)

   A) 0.6466 B) 0.6823 C) 0.7180 D) 0.7537 E) 0.7894
   F) 0.8251 G) 0.8608 H) 0.8965 I) 0.9322 J) 0.9679

Solution. *Answer:* ***H***

Let X be the annual cost of owning a dog and Y the annual cost of owning a cat. The annual cost of owning one dog and two cats is W = X + 2 Y. This is a normal random variable with E(W) = E(X) + 2 E(Y) = 695 + 2 (705) = 2105. The variance of W is Var(W) = Var(X) + 2^2 *Var(Y) =* $(45)^2 + 4\,(35)^2 = 6925$. *Thus,* $s_W = \sqrt{6925} = 83.21658$. *We have*

$$\begin{aligned} P(W > 2000) &= P\left(\frac{W-2105}{83.21658} > \frac{2000-2105}{83.21658}\right) \\ &= P(Z > -1.261768) \\ &= P(Z < 1.261768) \\ &= \Phi(1.261768) \\ &= 0.8964839. \end{aligned}$$

R code for the answer is

```
pnorm(2000, mean = 695+2*705, sd = sqrt( 45^2 + 4*35^2), lower.tail = FALSE).
```

5. Let Z_1 and Z_2 be independent standard normal random variables. Let $|Z_1|$ be the side length of one square. Let $|Z_2|$ be the side length of another square. Let A be the total area of the two squares. What is P(A > 2)?

   A) 0.3124 B) 0.3235 C) 0.3346 D) 0.3457 E) 0.3568
   F) 0.3679 G) 0.3790 H) 0.3901 I) 0.4012 J) 0.4123

Solution. *Answer:* ***F***

Because Z_1 *and* Z_2 *are independent standard normals, the sum* $A = Z_1^2 + Z_2^2$ *has chi-squared distribution with two degrees of freedom. We have* $P(A > 2) = P(\chi^2_2 > 2)$. *With software, such as the command*

```
pchisq(2, df = 2, lower.tail = FALSE)
```

in R, or something similar using a statistics calculator, we obtain 0.3678794 for the requested probability. If the given table is used instead, then we interpolate between the two values $(p, x) = (2.4079, 0.300)$ *and* $(p, x) = (1.8326, 0.400)$. *We find*

$$p = \frac{(0.300-0.400)}{(2.4079-1.8326)}\,(x-2.4079) + 0.300 = -0.1738224\,(x-2.4079) + 0.300.$$

For x = 2 we have $p = -0.1738224\,(2-2.4079) + 0.300 = 0.3709022$. *This differs from the software answer by only 0.0030228.*

6. To investigate the pattern of bomb hits in London during World War II, a rectangular grid was placed over the map of a 6 km × 6 km square area of London. The superimposed grid subdivided the area into 576 subsquares, each with a side length of 0.25 km. For each bomb that landed in the large square, the subsquare in which the bomb hit was noted. No subsquare was hit by 6 or more bombs. For each $k = 0, 1, 2, 3, 4, 5$, the number $N(k)$ of subsquares with exactly k hits was counted. The results are shown in the first two columns of the following table:

k	$N(k)$	$N(k)/576$
0	229	0.3976
1	211	0.3663
2	93	0.1615
3	35	0.0608
4	7	0.0122
5	1	0.0017
6+	0	0.0000
	576	1.0001

What was the average number of bomb hits per subsquare? (You will use this value as well as the third column of the table in the next question, which concerns the pattern of these bomb hits.)

A) 0.4770 B) 0.5523 C) 0.6276 D) 0.7029 E) 0.7782
F) 0.8535 G) 0.9288 H) 1.0041 I) 1.0794 J) 1.1547

Solution. *Answer:* ***G***

The number of bomb hits in the 6 *km*×6 *km large square was* $0 \times 229 + 1 \times 211 + 2 \times 93 + 3 \times 35 + 4 \times 7 + 5 \times 1$, *or 535. The average number of hits per subsquare was 535/576, or 0.9288.*

7. Were the bomb hits described in the preceding problem just inexplicable random scatter, or did the locations of the strikes fit some pattern? The empirical frequencies $N(k)/576$ found in the third column of the table given in the last problem constitute a probability distribution. (Their sum is not exactly 1 due to a small rounding error.) Let us try to model the empirical frequencies $N(k)/576$, $k = 0, 1, 2, 3, 4, 5, 6+$ by the probability function $\mathrm{P}(\mathrm{X} = k)$ of the Poisson r.v. X that has mean $\mathrm{E}(\mathrm{X})$ equal to the average number of bomb hits per subsquare, a number that was calculated in the preceding problem. What is the value $\mathrm{P}(\mathrm{X} = 2)$ that approximates the empirical frequency $N(2)/576 = 0.1615$?

A) 0.1515 B) 0.1542 C) 0.1569 D) 0.1596 E) 0.1623
F) 0.1650 G) 0.1677 H) 0.1704 I) 0.1731 J) 0.1758

Solution. *Answer:* ***H***

Let $\lambda = 0.9288$ *and let X be the Poisson r.v. with values* 0, 1, 2, 3, … *and probability function* $P(X = k) = \exp(-\lambda)\, \lambda^k/k!$ *The values of this p.f. can be found in the following table:*

k	$N(k)$	$N(k)/576$	$P(X=k)$
0	*229*	*0.3976*	*0.3950*
1	*211*	*0.3663*	*0.3669*
2	*93*	*0.1615*	*0.1704*
3	*35*	*0.0608*	*0.0528*
4	*7*	*0.0122*	*0.0122*
5	*1*	*0.0017*	*0.0023*
6+	*0*	*0.0000*	*0.0004*
	576	*1.0001*	*1.0000*

8. The number of tornadoes in Texas in November is a Poisson random variable with mean 5.03. Assuming that the numbers of tornadoes in different years are independent, what, approximately, is the probability that in 30 consecutive years Texas has more than 160 November tornadoes? Use the normal approximation with correction for continuity. Note: The variance of a Poisson r.v. is equal to its mean.

   A) 0.0547 B) 0.1360 C) 0.2173 D) 0.2986 E) 0.3799
   F) 0.4612 G) 0.5425 H) 0.6238 I) 0.7051 J) 0.7864

Solution. *Answer:* ***C***

Let X_j be the number of November tornadoes in the j^{th} year. We calculate the mean and variance of $X_1 + \cdots + X_{30}$ to be 30×5.03*, or 150.9. Thus,*

$$\begin{aligned} P(X_1 + \cdots + X_{30} > 160) &= P(X_1 + \cdots + X_{30} \geq 161) \\ &\approx P\left(N\left(150.9, \sqrt{150.9}\right) \geq 160.5\right) \\ &= P\left(\frac{N\left(150.9\sqrt{150.9}\right) - 150.9}{\sqrt{150.9}} \geq \frac{160.5 - 150.9}{\sqrt{150.9}}\right) \\ &= P(Z \geq 0.7814957) \\ &= 1 - \Phi(0.7814957) \\ &= 0.2172555. \end{aligned}$$

The R code
```
pnorm(160.5, mean = 30*5.03, sd= sqrt(30*5.03), lower.tail=FALSE)
```
returns 0.2172555, which is the answer to this problem. The R code
```
1-sum(dpois(0:160, 30*5.03))
```
evaluates the exact value as 0.2156821. The error of our approximation is 0.73%.

9. According to the National Institutes of Health, the proportion of American adults age 20 and older who are obese is 0.357. In a random sample of 100 adult Americans, what, approximately, is the probability that fewer than 30 of those sampled are obese? Do not use the correction for continuity in this problem: you will be asked to use it in the next.

   A) 0.0397 B) 0.0810 C) 0.1223 D) 0.1636 E) 0.2049
   F) 0.2462 G) 0.2875 H) 0.3288 I) 0.3701 J) 0.4114

Solution. *Answer:* ***B***

The exact probability is $\sum_{k=0}^{29} \binom{100}{k} (0.357)^k (0.643)^{100-k}$*, or 0.0965. R code for this exact probability is* `sum(dbinom(0:29, 100, 0.357))`.

But exactness was not called for. Let us proceed to the requested approximation. Let $X_j = 1$ if the j^{th} samplee is obese (BMI at least 30), and let $X_j = 0$ otherwise. For the calculation that follows, we calculate $\sqrt{100 \times 0.357 \times 0.643} = 4.791148$. The normal approximation without the correction for continuity is

$$
\begin{aligned}
&P(X_1 + \cdots + X_{100} < 30) \\
&\qquad = P(X_1 + \cdots + X_{100} \leq 29) \\
&\qquad \approx P\big(N\big(100 \times 0.357, \sqrt{100 \times 0.357 \times 0.643}\big) \leq 29\big) \\
&\qquad = P\left(\frac{N(35.7, 4.791148) - 35.7}{4.791148} \leq \frac{29 - 35.7}{4.791148}\right) \\
&\qquad = P(Z \leq -1.398412) \\
&\qquad = \Phi(-1.398412) \\
&\qquad = 1 - \Phi(1.398412) \\
&\qquad = 0.0809947.
\end{aligned}
$$

The above evaluations were obtained using R. A direct R calculation of the answer would be obtained by the single command
`pnorm(29, mean = 100*0.357, sd = sqrt(100*0.357*0.643))`.

Without technology, using a table to calculate $\Phi(1.398412)$, we obtain

$$
\begin{aligned}
\Phi(1.398412) &= \Phi(1.39 + 0.008412) \\
&= \Phi\left(1.39 + \frac{8412}{10000} \times 0.001\right) \\
&= \Phi\left(1.39 + \frac{8412}{10000} \times (1.40 - 1.39)\right) \\
&\approx \Phi(1.39) + \frac{8412}{10000} \times \big(\Phi(1.40) - \Phi(1.39)\big) \\
&= 0.9177 + \frac{8412}{10000} \times \big(0.9192 - 0.9177\big) \\
&= 0.9189618.
\end{aligned}
$$

The answer is 1 - 0.9189618, or 0.0810382. Whether a calculator, software, or a table is used, the answer rounds to 0.0810. We note that the approximation is ballpark, but not especially accurate.

10. As stated in the preceding problem, the proportion of American adults age 20 and older who are obese is 0.357. In a random sample of 100 adult Americans age 20 and older, what, approximately, is the probability that fewer than 30 of those sampled are obese? Use the normal approximation with correction for continuity in this problem.

A) 0.0376	B) 0.0677	C) 0.0978	D) 0.1279	E) 0.1580
F) 0.1881	G) 0.2182	H) 0.2483	I) 0.2784	J) 0.3085

Solution. *Answer:* ***C***

Recall from the last problem that the exact answer to four decimal places is 0.0965 and the normal approximation without the correction for continuity is 0.0810. Let us proceed to the requested normal approximation with continuity correction. Let $X_j = 1$ if the j^{th} samplee is obese, and let $X_j = 0$ otherwise. For the calculation *that follows, we calculate* $\sqrt{100 \times 0.357 \times 0.643} = 4.791148$. *The normal approximation* with *the correction for continuity is*

$$\begin{aligned}
& P(X_1 + \cdots + X_{100} < 30) \\
&= P(X_1 + \cdots + X_{100} \le 29) \\
&\approx P\left(N\left(100 \times 0.357, \sqrt{100 \times 0.357 \times 0.643}\right) \le 29.5\right) \\
&= P\left(\frac{N(35.7, 4.791148) - 35.7}{4.791148} \le \frac{(29.5 - 35.7)}{4.791148}\right) \\
&= P(Z \le -1.294053) \\
&= \Phi(-1.294053) \\
&= 1 - \Phi(1.294053) \\
&= 0.09782356.
\end{aligned}$$

The above evaluations were obtained in R. Using a table to calculate $\Phi(1.294053)$, *we obtain*

$$\begin{aligned}
\Phi(1.294053) &= \Phi(1.29 + 0.004053) \\
&= \Phi\left(1.29 + \frac{4053}{10000} \times 0.001\right) \\
&= \Phi\left(1.29 + \frac{4053}{10000} \times (1.30 - 1.29)\right) \\
&\approx \Phi(1.29) + \frac{4053}{10000} \times \left(\Phi(1.30) - \Phi(1.29)\right) \\
&= 0.9015 + \frac{4053}{10000} \times \left(0.9032 - 0.9015\right) \\
&= 0.902189.
\end{aligned}$$

The answer is 1 - 0.902189, or 0.097811. Whether a calculator, software, or a table is used, the answer rounds to 0.0978. We note that this approximation is rather accurate, being off the mark by only 0.0013. The one-line R code that answers this problem is
`pnorm(29.5, mean = 100*0.357, sd = sqrt(100*0.357*0.643)).`

11. Suppose that for a certain subpopulation of American adults, BMI is normally distributed with mean 26.65 and standard deviation 2.5. (The mean BMI for the entire population of American adults is 26.65 but the normal model does not fit the entire population very well.) If one member of the subpopulation was selected, the probability of the samplee's BMI being between 26.3 and 27 would be 0.11134. Let us contrast this probability with that of a larger sample. If a random sample of size 100 were drawn from the subpopulation, then what would be the probability of the average BMI of the samplee's being between 26.3 and 27?

A) 0.1417	B) 0.2288	C) 0.3159	D) 0.4030	E) 0.4901
F) 0.5772	G) 0.6643	H) 0.7514	I) 0.8385	J) 0.9256

Solution. *Answer:* ***I***

The R code for the given probability for one samplee is
`pnorm(27, mean = 26.65, sd = 2.5) - pnorm(26.3, mean = 26.65, sd = 2.5).`
For a random sample of size 100, the appropriate R code would be

```
pnorm(27, mean = 26.65, sd = 2.5/sqrt(100)) - pnorm(26.3, mean = 26.65, sd = 2.5/sqrt(100)).
```

Without technology, using the given Phi table, we calculate

$$\begin{aligned} P\left(26.3 \le \overline{X} \le 27\right) &= P\left(\frac{(26.3-26.65)}{2.5/sqrt100} \le \frac{\overline{X}-26.65}{2.5/sqrt100} \le \frac{(27-26.65)}{2.5/sqrt100}\right) \\ &= P(-1.4 \le Z \le 1.4) \\ &= \Phi(1.4) - \Phi(-1.4) \\ &= 2\,\Phi(1.4) - 1 \\ &= 2\,(0.9192) - 1 \\ &= 0.8385. \end{aligned}$$

12. We continue with the subpopulation of American adults for whom BMI is normally distributed with mean 26.65 and standard deviation 2.5. If a random sample of size 101 is drawn from that subpopulation, then what is the probability that the the sample variance is greater than 5.988?

A) 0.400 B) 0.500 C) 0.600 D) 0.700 E) 0.750
F) 0.800 G) 0.900 H) 0.950 I) 0.975 J) 0.990

Solution. *Answer:* ***C***

Let S^2 *be the sample variance. Then* $\frac{(101-1)}{(2.5)^2}\cdot S^2 \sim \chi^2_{101-1}$*. We have*

$$\begin{aligned} P\left(S^2 > 5.99\right) &= P\left(\frac{(101-1)}{(2.5)^2}\cdot S^2 > \frac{(101-1)}{(2.5)^2}\cdot 5.988\right) \\ &= P\left(\chi^2_{100} > 95.808\right) \\ &= 0.600. \end{aligned}$$

R code that answers this problem is

```
pchisq((101-1)*5.988/(2.5)^2, df=100, lower.tail=FALSE) .
```

13. Hemoglobin is the oxygen-transport protein in red blood cells. If a random sample of 81 adult women results in an observed sample mean of 13.58 grams per deciliter and an observed sample standard deviation of 0.893 grams per deciliter, what is the probability that the true mean blood hemoglobin level for adult women is greater than 13.75?

A) 0.0453 B) 0.0580 C) 0.0707 D) 0.0834 E) 0.0961
F) 0.1088 G) 0.1215 H) 0.1342 I) 0.1469 J) 0.1596

Solution. *Answer:* ***A***

We use $\left(\overline{X}-\mu\right)/(S/\sqrt{81}) \sim t_{81-1}$*. It follows that*

$$\begin{aligned} P(\mu > 13.75) &= P(-\mu < -13.75) \\ &= P(\overline{X}-\mu < \overline{X}-13.75) \\ &= P\left(\frac{\overline{X}-\mu}{S/\sqrt{81}} < \frac{\overline{X}-13.75}{S/\sqrt{81}}\right) \\ &= P\left(t_{80} < \frac{13.58-13.75}{0.893/\sqrt{81}}\right) \\ &= P(t_{80} < -1.713326) \\ &= P(t_{80} > 1.713326) \\ &= 0.04526. \end{aligned}$$

The value in the preceding line was obtained using R: `pt(1.713326, df = 80, lower.tail=FALSE)`*. Using the given table, we can approximate this value by using the df = 80 line of the table to obtain the points* $(x, p) = (1.6620, 0.50)$ *and* $(x, p) = (1.9867, 0.025)$*, and interpolating between them:*

$$p = \frac{(0.050 - 0.025)}{(1.6620 - 1.9867)}(x - 1.6620) + 0.050 = -0.07699415\,(x - 1.6620) + 0.050.$$

For $x = 1.713326$ *we have* $p = -1.462889\,(1.713326 - 1.6620) + 0.050 = 0.0460482$*, which differs from the software value by only 0.0008. R code for the answer to this problem is*
`pt( (13.58-13.75)/(0.893/sqrt(81)), df = 81-1)`.

14. According to a Harris Interactive survey of 2300 American workers, 20% are contacted by their boss when they are on vacation (as reported by MarketWatch, 11 September 2015). If this survey had been used to obtain a 95% confidence interval for the proportion of American workers on vacation contacted by a boss, what would the margin of error have been?

A) 0.00828	B) 0.01635	C) 0.02442	D) 0.03249	E) 0.04056
F) 0.04863	G) 0.05670	H) 0.06477	I) 0.07284	J) 0.08091

Solution. *Answer:* ***B***

The requested margin of error is

$$z_{0.05/2}\sqrt{\frac{(0.20)(0.80)}{2300}} = 1.959964 \cdot 0.008340577 = 0.01634723.$$

R code that gives the answer is `qnorm(0.975)*sqrt(0.20*0.80/2300)`.

15. Ripped from the headlines! According to a CNN/ORC poll of 1001 adult Americans, two-thirds want the Senate to hold confirmation hearings on the candidacy of Merrick Garland, President Barack Obama's nominee for filling the vacant seat on the Supreme Court. CNN's article mentions that the poll was conducted between 17 March 2016 and 20 March 2016. The report also mentions that the margin of error is plus or minus 3 percentage points. However, the article omits any mention of the confidence level, a crucial component of interval estimation without which the reported numbers are essentially meaningless. What was the confidence level?

A) 94.8813%	B) 95.0000%	C) 95.1187%	D) 95.2374%	E) 95.3561%
F) 95.4748%	G) 95.5935%	H) 95.7122%	I) 95.8309%	J) 95.9496%

Solution. *Answer:* ***G***

The headline was, "Support for SCOTUS hearings remains strong, CNN/ORC poll finds", `http://www.cnn.com/2016/03/25/politics/merrick-garland-supreme-court-nominee/`*, Retrieved: 31 March 2016.*

Let's start by calculating the standard error:

$$SE(\widehat{p}) = \sqrt{\frac{(2/3)(1 - 2/3)}{1001}} = 0.01489967.$$

Expressed in terms of an unknown critical value $z_{\alpha/2}$*, the margin of error is given by* $ME(\widehat{p}) = z_{\alpha/2}\,SE(\widehat{p}) = z_{\alpha/2} \times 0.01489967$*. Given that the margin of error is 0.03, we have* $z_{\alpha/2} = 0.03/0.01489967 = 2.013467$*. We find* $P(Z > 2.013467) = 0.02203276$*, or* $\alpha/2 = 0.02203276$*, or* $\alpha = 0.04406552$*, which means that the confidence level is* $100\,(1 - 0.04406552)\%$*, or 95.5935% R code that answers this problem is*
`100*(1-2*pnorm(0.03/sqrt( (2/3)*(1/3)/1001 ), lower.tail = FALSE))`.

16. In November 2015 Gallup released the results of a survey concerning American adults who had had a heart attack. The report is filled with interesting statistics. For example, West Virginia is the state with the highest heart attack rate (7.7%) and Utah the lowest (2.4%). What is especially interesting for us in this problem is that Gallup reported that the margin of error in the survey was 1.08% at the 95% confidence level. Note: Gallup reported all proportions in the form of percentages rather than probabilities. In order to attain the stated margin of error at the 95% confidence level, what would the smallest sample size have been?

A) 5858 B) 6122 C) 6386 D) 6650 E) 6914
F) 7178 G) 7442 H) 7706 I) 7970 J) 8234

Solution. *Answer:* ***J***

Here $z_{0.25} = 1.959964$ *and* $2\,ME_0 = 2 \times 0.0108 = 0.0216$. *The required smallest sample size is* $n = \lceil (1.959964/0.0216)^2 \rceil = \lceil 8233.58 \rceil = 8234$ *rounded up to the nest whole number.*

17. The overall incidence of *anesthesia awareness*, also referred to as *unintended intra-operative awareness*, is difficult to pin down, but the phenomenon has been investigated for several particular surgical procedures. In one study, 37 patients who were continuously or almost continuously given an anesthetic for endotracheal intubation during surgery were interviewed after recovery. Of these 37 patients, 4 recalled their surgery (Bogetz MS and Katz JA, *Recall of surgery for major trauma*, Anesthesiology, 1984). Find the lower endpoint of a 95% confidence interval for the population proportion of patients who have an awareness of their surgery when given an anesthetic for endotracheal intubation.

A) 0.0046 B) 0.0158 C) 0.0270 D) 0.0382 E) 0.0494
F) 0.0606 G) 0.0718 H) 0.0830 I) 0.0942 J) 0.1054

Solution. *Answer:* ***D***

The 10-10 Success-Failure Condition is not satisfied. Therefore, the Agresti-Coull adjustment is needed. With the four phony trials included, we have $n' = 37 + 4 = 41$, $n'_S = 4 + 2 = 6$, $n'_F = 33 + 2 = 35$, $\widehat{p}' = 6/41 = 0.1463415$, $\widehat{q}' = 1 - \widehat{p}' = 1 - 0.1463415 = 0.8536585$ *(or, alternatively, 35/41),* $SE(\widehat{p}') = \sqrt{(0.1463415)(0.8536585)/41} = 0.05519934$, $ME(\widehat{p}') = z_{0.025} \times SE(\widehat{p}') = (1.959964)(0.05519934) = 0.1081887$, *and* $\widehat{p}' \pm ME(\widehat{p}')$ *is the interval* $[0.0381528, 0.2545302]$. *R code for the answer is*

```
6/41 - qnorm(0.975)*sqrt((6/41)*(35/41)/41).
```

18. Alums of Baron Byng High School have included a renowned mathematician, a Nobelist in chemistry, a prize-winning novelist, and an enterprising overactor who is out of this world. A principal of the high school undertook to compare the IQ scores of the school's students with the IQ scores of the general population. A random sample of 334 of the school's students led to the conclusion that the school standard deviation was the same as that for the general population, namely 15. The observed sample mean of 102.13 was, however, slightly greater than the general population mean of 100. Does this suggest that the true school mean μ was greater than 100? Constructing a 99% confidence interval can shed light on this question. If the lower limit of the confidence interval for μ is greater than 100, then we can be very confident that $\mu > 100$. What is the lower endpoint of a 99% confidence interval for μ?

A) 99.8401 B) 99.9280 C) 100.0159 D) 100.1038 E) 100.1917
F) 100.2796 G) 100.3675 H) 100.4554 I) 100.5433 J) 100.6312

Solution. *Answer:* ***C***

Louis Nirenberg, Rudolph Marcus, Mordecai Richler, and Captain James T. Kirk—all Baron Byngies. Because $n = 334$, $\widehat{\mu}_{BBHS} = 102.13$, $\sigma = 15$, *and* $z_{0.005} = 2.575829$, *the required lower bound is* $\widehat{\mu}_{BBHS} - z_{0.005}\,\sigma/\sqrt{n}$, *or* $102.13 - 2.575829 \times 15/\sqrt{334}$, *or 100.0159.*

19. In everyday life, there is scant evidence that humans are getting more intelligent. Nevertheless, since about 1930, there has been a steady, year-by-year increase in the average IQ for the general population. This phenomenon has become known as the *Flynn effect* after James R. Flynn, who documented it. To estimate the magnitude of the Flynn effect, a researcher had 64 randomly selected adults (from the population of 20-year-olds) complete a 10-year-old IQ exam. The old IQ exam had been "standardized" to yield a mean of 100 and standard deviation of 15 when it was originally given ten years before the experiment. In the experiment, the average score was 107 with a sample standard deviation of 12. What was the margin of error of a 80% confidence interval for the mean score of a 20-year-old taking the 10-year-old IQ exam. Note: Although the exam was design to yield a normal distribution of scores when it was first administered, do not assume that it produced a normal distribution of scores 10 years later.

A) 1.8347	B) 1.8566	C) 1.8785	D) 1.9004	E) 1.9223
F) 1.9442	G) 1.9661	H) 1.9880	I) 2.0099	J) 2.0318

Solution. *Answer:* ***E***

We do not know either μ *or* σ. *We also are not to assume that the distribution of scores is normal. However,* $n = 64$ *is sufficiently large to allow us to approximate* $\overline{X}$ *with a normal r.v. We have* $\widehat{\mu} = 107, S = 12$, *and we look up* $z_{0.10} = 1.281552$. *The margin of error is* $z_{0.10}\, S/\sqrt{n}$, *or* $1.281552 \times 12/\sqrt{64}$, *or 1.922328.*

20. Specimens were taken from 10 individuals to determine the percentage X of calcium in the composition of healthy teeth. The observed sample mean was 35.6478 and the observed sample variance was 0.5166. What is the upper limit of a 99% confidence interval for μ_X? Assume that X is normally distributed.

A) 36.2793	B) 36.3864	C) 36.4935	D) 36.6006	E) 36.7077
F) 36.8148	G) 36.9219	H) 37.0290	I) 37.1361	J) 37.2432

Solution. *Answer:* ***B***

For this small sample ($n = 10$*) from a normal distribution with unknown standard deviation, we must use a Student-t distribution with* $n-1$, *or 9, degrees of freedom. From the Student-t table, we have* $t_{0.005,9} = 3.2498$. *The sample standard deviation is* $S = \sqrt{0.5166} = 0.7187489$. *The standard error is* $S/\sqrt{n}$, *or* $0.7187489/\sqrt{10}$, *or 0.2272884. The margin of error ME is* 3.2498×0.2272884, *or 0.7386418. The upper limit of the 99% confidence interval is* $\overline{X} + ME$, *or* $35.6478 + 0.7386418$, *or 36.38645. R code for the answer to this problem is*
`35.6478 + qt(0.995, df = 9)*sqrt(0.5166/10)`

21. In a study of the effect of humor on interpersonal relationships, 60 female undergraduates who identified themselves as heterosexual were shown a photograph of "James". The women were informed that James was single, ambitious, and that he had good job prospects. In this problem we are interested in the subgroup of 30 women who were also told that, "One person who knows James well said, 'I have known James a long time and he has a great sense of humor.'" The women were asked to rank James's attractiveness X on a Likert-type scale from 1 (very unattractive) to 7 (very attractive). The sample mean was 4.53 with sample standard deviation 1.04. Using a z-score, find the length of a 95% confidence interval for μ_X. (Taken from Elizabeth McGee and Mark Shevlin, *Effect of Humor on Interpersonal Attraction and Mate Selection*, The Journal of Psychology, **143(1)**, 2009, 6777.) The next problem continues with this humorous study.

A) 0.4838	B) 0.5359	C) 0.5880	D) 0.6401	E) 0.6922
F) 0.7443	G) 0.7964	H) 0.8485	I) 0.9006	J) 0.9527

Solution. *Answer:* ***F***

For this sample, we have ($n = 30$) and we may use a normal approximation. We look up $z_{0.025} = 1.959964$. *The sample standard deviation is $S = 1.04$. The standard error is $S/\sqrt{n}$, or* $1.04/\sqrt{30}$, *or 0.1898772. The margin of error is* $1.959964 \times 0.1898772$, *or 0.3721525. The length of the 95% confidence interval is twice the margin of error, or 0.744305. R code for the answer is* `2*qnorm(0.975)*1.04/sqrt(30)`.

22. In the study described in the preceding problem, 30 females composing a second group of heterosexual undergraduates were shown the same photo of "James". As with the women in the first group of 30 women, the participants in the second group were told that James was single, ambitious, and that he had good job prospects. However, the women in the second group did *not* receive a testimonial of James's good sense of humor. Instead they were told that, "One person who knows James well said, 'I have known James a long time and and I can say that in relation to his sense of humor—he doesn't have one.'" The women were asked to rank the attractiveness Y of the humorless James on a Likert-type scale from 1 (very unattractive) to 7 (very attractive). The sample mean was 3.30 with sample standard deviation 1.18. What is the lower endpoint of a 95% confidence interval for the difference $\mu_{\mathrm{X}} - \mu_{\mathrm{Y}}$ of means?. (You will note that if this lower bound is positive, then we can be 95% sure that heterosexual female undergraduates find the humourful James more attractive than the humourless James.)

A) -0.2834 B) -0.1476 C) -0.0118 D) 0.1240 E) 0.2598
F) 0.3956 G) 0.5314 H) 0.6672 I) 0.8030 J) 0.9388

Solution. *Answer:* ***H***

Let $n = m = 30$ denote the sample sizes. The required lower endpoint of a 95% confidence interval for $\mu_X - \mu_Y$ is

$$\begin{aligned} \overline{X} - \overline{Y} &- z_{0.025} \times \sqrt{\frac{S_X^2}{n} + \frac{S_Y^2}{m}} \\ &- 4.53 - 3.30 - 1.959964\sqrt{\frac{1.04^2}{30} + \frac{1.18^2}{30}} \\ &- 0.6671569. \end{aligned}$$

R code for the solution is
`4.53 - 3.30 - qnorm(0.975)*sqrt( (1.04)^/30 + (1.18)^/30).`

23. Hypersexual disorder is a recognized complication of craniocerebral trauma. Initial treatment is generally non-pharmacological (psychotherapy), but chemical treatment is an option, and medroxyprogesterone acetate (MPA) is a drug that is often used. In a clinical study, young males with hypersexual behavior following traumatic brain injury were treated weekly with 400 mg of MPA for six months. The testosterone level (in nanograms per deciliter) of each patient was measured before and after treatment. To reduce the amount of routine computation, four cases have been selected:

Patient	Before	After
1	849	96
2	890	31
3	362	46
4	1006	113

(Data from Emory, Cole, and Meyer, *Journal of Head Trauma Rehabilitation*, 1995) What is the lower endpoint of a 90% confidence interval for the mean *decrease* in testosterone level following treatment? The number we seek is positive because we have asked for the decrease, not the change. Assume that testosterone levels are normally distributed before and after treatment.

A) 391.95 B) 393.66 C) 395.38 D) 397.10 E) 398.82
F) 400.53 G) 402.25 H) 403.97 I) 405.68 J) 407.40

Solution. *Answer:* ***A***

Let X and Y denote the testosterone levels before treatment and after. The differences W = X - Y are $Y_1 - X_1 = 849 - 96 = 753$, $Y_2 - X_2 = 890 - 31 = 859$, $Y_3 - X_3 = 362 - 46 = 316$, $Y_4 - X_4 = 1006 - 113 = 893$. *We calculate* $\overline{W} = 705.25$ *and* $S = Sd(W) = 266.261$. *The lower endpoint of a 90% confidence interval for the mean decrease is*

$$\overline{W} + t_{0.95,3}\,\frac{S}{\sqrt{4}} = 705.25 - 2.353363 \times \frac{266.261}{2} = 391.9456.$$

R code for the answer:

```
> D = c(849-96, 890-31, 362-46, 1006-113); mean(D)-qt(0.95, df = 4-1)*sd(D)/sqrt(4).
```

24 The testosterone level of an unmedicated 20-30 year-old male is normally distributed with mean 670 ng/dL and unknown variance. To test the hypothesis that the population variance σ^2 is 160^2, a 95% confidence interval for the variance is constructed using a random sample of men in the population. To reduce tedious arithmetic, let us suppose that the sample consists only of the four measurements 260, 675, 705, and 915. What is the standard deviation that corresponds to the lower bound of the confidence interval for the variance? (All the answer choices—including the correct answer—are less than 160. If the correct answer was greater than 160, then we'd be quite confident that the hypothesis $\sigma^2 = 160^2$ is false.)

A) 146.0671 B) 147.6061 C) 149.1451 D) 150.6841 E) 152.2231
F) 153.762 G) 155.3011 H) 156.8401 I) 158.3791 J) 159.9181

Solution. *Answer:* ***G***

In this problem $n = 4$, *the sample variance* S^2 *is computed to be 75156.25, and the confidence level is* $\alpha = 0.05$. *We look up* $\chi^2_{1-\alpha/2,n-1} = \chi^2_{0.975,3} = 9.348404$. *A* $100(1-\alpha)\%$ *confidence interval is*

$$\left[\frac{n-1}{\chi^2_{\alpha/2,n-1}}\,S^2, \frac{n-1}{\chi^2_{1-\alpha/2,n-1}}\,S^2\right].$$

The required lower bound is $(n-1)\,S^2/\chi^2_{1-\alpha/2,n-1}$, *or* $3\,(75156.25)/9.348404$, *or 24118.42. The standard deviation corresponding to this variance is its square root: 155.3011. R code for the answer is*

```
X = c(260, 675, 705, 915); sqrt(3*var(X)/qchisq(0.025, df = 3, lower.tail = FALSE)).
```

25. In a study of 11 edematous patients, 7 were randomly selected and given a diuretic agent. The other 4 were given a placebo. Urine sodium concentrations were measured (in meq/L) 24 hours later. Let X be the measurement of a patient who receives the diuretic agent and let Y be the measurement of a patient who receives the placebo. The observed sample means and sample standard deviations for the concentrations were $\overline{X} = 41.4286$, $\overline{Y} = 16.6750$ and $S_X = 26.5009$, $S_Y = 7.7852$. Find the upper endpoint of a 95% confidence interval for the difference $\mu_X - \mu_Y$ of the true means. Assume normality of the distributions, but be conservative: Do not assume that $\sigma_X = \sigma_Y$.

A) 52.3120 B) 53.1421 C) 53.9722 D) 54.8023 E) 55.6324
F) 56.4625 G) 57.2926 H) 58.1227 I) 58.9528 J) 59.7829

Solution. *Answer:* ***I***

Let $n = 7$ *and* $m = 4$ *denote the sample sizes. The required upper endpoint of a 95% confidence interval for* $\mu_X - \mu_Y$ *is*

$$\overline{X} - \overline{Y} + t_{\alpha/2,df} \times \sqrt{\frac{S_X^2}{n} + \frac{S_Y^2}{m}}$$

where df is the minimum of $n - 1$ *and* $m - 1$*, namely 3. The required value is* $41.4286 - 16.6750 + t_{0.025,3}\sqrt{(26.5009)^2/7 + (7.7852)^2/4}$*, or* $41.4286 - 16.675 + 34.19917$*, or 58.95277.*
R code for the answer is

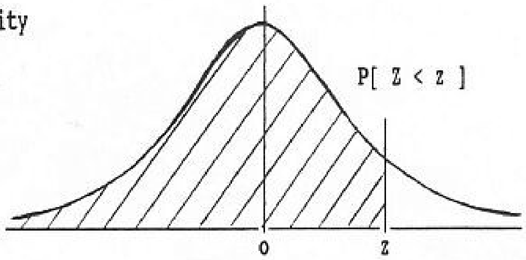
```
41.4286-16.6750+qt(0.975,3)*sqrt(26.5009^2/7 + 7.7852^2/4).
```

STANDARD STATISTICAL TABLES

1. Areas under the Normal Distribution

The table gives the cumulative probability up to the standardised normal value z i.e.

$$P[\, Z < z \,] = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp(-\tfrac{1}{2}Z^2)\, dZ$$

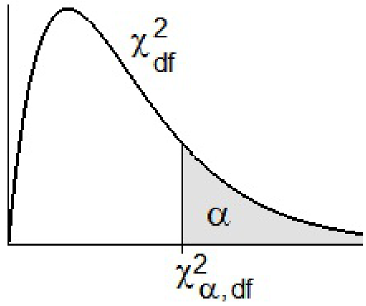


z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7020	0.7054	0.7089	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8804	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9865	0.9868	0.9871	0.9874	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9924	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9980	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986

z	3.00	3.10	3.20	3.30	3.40	3.50	3.60	3.70	3.80	3.90
P	0.9986	0.9990	0.9993	0.9995	0.9997	0.9998	0.9998	0.9999	0.9999	1.0000

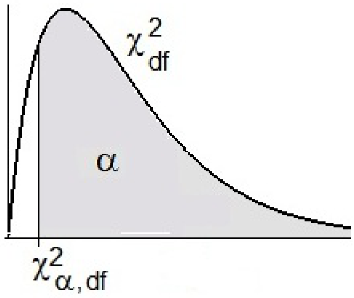
Table of $\Phi(z)$ for Nonnegative z

Values of $\chi^2_{\alpha,df}$ $\quad P(\chi^2_{df} \geq \chi^2_{\alpha,df}) = \alpha$



df \ α	0.005	0.010	0.025	0.050	0.100	0.200	0.250	0.300	0.400	0.500
1	7.8794	6.6349	5.0239	3.8415	2.7055	1.6424	1.3233	1.0742	0.7083	0.4549
2	10.5966	9.2103	7.3778	5.9915	4.6052	3.2189	2.7726	2.4079	1.8326	1.3863
3	12.8382	11.3449	9.3484	7.8147	6.2514	4.6416	4.1083	3.6649	2.9462	2.3660
4	14.8603	13.2767	11.1433	9.4877	7.7794	5.9886	5.3853	4.8784	4.0446	3.3567
5	16.7496	15.0863	12.8325	11.0705	9.2364	7.2893	6.6257	6.0644	5.1319	4.3515
6	18.5476	16.8119	14.4494	12.5916	10.6446	8.5581	7.8408	7.2311	6.2108	5.3481
7	20.2777	18.4753	16.0128	14.0671	12.0170	9.8032	9.0371	8.3834	7.2832	6.3458
8	21.9550	20.0902	17.5345	15.5073	13.3616	11.0301	10.2189	9.5245	8.3505	7.3441
9	23.5894	21.6660	19.0228	16.9190	14.6837	12.2421	11.3888	10.6564	9.4136	8.3428
10	25.1882	23.2093	20.4832	18.3070	15.9872	13.4420	12.5489	11.7807	10.4732	9.3418
11	26.7568	24.7250	21.9200	19.6751	17.2750	14.6314	13.7007	12.8987	11.5298	10.3410
12	28.2995	26.2170	23.3367	21.0261	18.5493	15.8120	14.8454	14.0111	12.5838	11.3403
13	29.8195	27.6882	24.7356	22.3620	19.8119	16.9848	15.9839	15.1187	13.6356	12.3398
14	31.3193	29.1412	26.1189	23.6848	21.0641	18.1508	17.1169	16.2221	14.6853	13.3393
15	32.8013	30.5779	27.4884	24.9958	22.3071	19.3107	18.2451	17.3217	15.7332	14.3389
16	34.2672	31.9999	28.8454	26.2962	23.5418	20.4651	19.3689	18.4179	16.7795	15.3385
17	35.7185	33.4087	30.1910	27.5871	24.7690	21.6146	20.4887	19.5110	17.8244	16.3382
18	37.1565	34.8053	31.5264	28.8693	25.9894	22.7595	21.6049	20.6014	18.8679	17.3379
19	38.5823	36.1909	32.8523	30.1435	27.2036	23.9004	22.7178	21.6891	19.9102	18.3377
20	39.9968	37.5662	34.1696	31.4104	28.4120	25.0375	23.8277	22.7745	20.9514	19.3374
21	41.4011	38.9322	35.4789	32.6706	29.6151	26.1711	24.9348	23.8578	21.9915	20.3372
22	42.7957	40.2894	36.7807	33.9244	30.8133	27.3015	26.0393	24.9390	23.0307	21.3370
23	44.1813	41.6384	38.0756	35.1725	32.0069	28.4288	27.1413	26.0184	24.0689	22.3369
24	45.5585	42.9798	39.3641	36.4150	33.1962	29.5533	28.2412	27.0960	25.1063	23.3367
25	46.9279	44.3141	40.6465	37.6525	34.3816	30.6752	29.3389	28.1719	26.1430	24.3366
30	53.6720	50.8922	46.9792	43.7730	40.2560	36.2502	34.7997	33.5302	31.3159	29.3360
40	66.7660	63.6907	59.3417	55.7585	51.8051	47.2685	45.6160	44.1649	41.6222	39.3353
50	79.4900	76.1539	71.4202	67.5048	63.1671	58.1638	56.3336	54.7228	51.8916	49.3349
60	91.9517	88.3794	83.2977	79.0819	74.3970	68.9721	66.9815	65.2265	62.1348	59.3347
70	104.2149	100.4252	95.0232	90.5312	85.5270	79.7146	77.5767	75.6893	72.3583	69.3345
80	116.3211	112.3288	106.6286	101.8795	96.5782	90.4053	88.1303	86.1197	82.5663	79.3343
90	128.2989	124.1163	118.1359	113.1453	107.5650	101.0537	98.6499	96.5238	92.7614	89.3342
100	140.1695	135.8067	129.5612	124.3421	118.4980	111.6667	109.1412	106.9058	102.9459	99.3341

Chi-Squared Values—Right Tails.



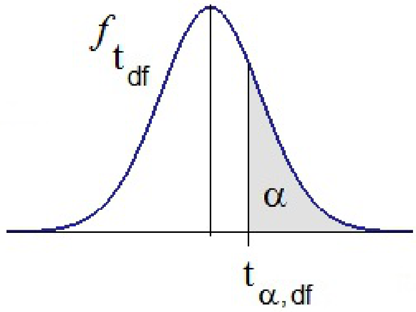
Values of $\chi^2_{\alpha,df}$ $\quad P(\chi^2_{df} \geqslant \chi^2_{\alpha,df}) = \alpha$

df \ α	0.600	0.700	0.750	0.800	0.900	0.950	0.975	0.990	0.995
1	0.2750	0.1485	0.1015	0.0642	0.0158	0.0039	0.0010	0.0002	0.0000
2	1.0217	0.7133	0.5754	0.4463	0.2107	0.1026	0.0506	0.0201	0.0100
3	1.8692	1.4237	1.2125	1.0052	0.5844	0.3518	0.2158	0.1148	0.0717
4	2.7528	2.1947	1.9226	1.6488	1.0636	0.7107	0.4844	0.2971	0.2070
5	3.6555	2.9999	2.6746	2.3425	1.6103	1.1455	0.8312	0.5543	0.4117
6	4.5702	3.8276	3.4546	3.0701	2.2041	1.6354	1.2373	0.8721	0.6757
7	5.4932	4.6713	4.2549	3.8223	2.8331	2.1673	1.6899	1.2390	0.9893
8	6.4226	5.5274	5.0706	4.5936	3.4895	2.7326	2.1797	1.6465	1.3444
9	7.3570	6.3933	5.8988	5.3801	4.1682	3.3251	2.7004	2.0879	1.7349
10	8.2955	7.2672	6.7372	6.1791	4.8652	3.9403	3.2470	2.5582	2.1559
11	9.2373	8.1479	7.5841	6.9887	5.5778	4.5748	3.8157	3.0535	2.6032
12	10.1820	9.0343	8.4384	7.8073	6.3038	5.2260	4.4038	3.5706	3.0738
13	11.1291	9.9257	9.2991	8.6339	7.0415	5.8919	5.0088	4.1069	3.5650
14	12.0785	10.8215	10.1653	9.4673	7.7895	6.5706	5.6287	4.6604	4.0747
15	13.0297	11.7212	11.0365	10.3070	8.5468	7.2609	6.2621	5.2293	4.6009
16	13.9827	12.6243	11.9122	11.1521	9.3122	7.9616	6.9077	5.8122	5.1422
17	14.9373	13.5307	12.7919	12.0023	10.0852	8.6718	7.5642	6.4078	5.6972
18	15.8932	14.4399	13.6753	12.8570	10.8649	9.3905	8.2307	7.0149	6.2648
19	16.8504	15.3517	14.5620	13.7158	11.6509	10.1170	8.9065	7.6327	6.8440
20	17.8088	16.2659	15.4518	14.5784	12.4426	10.8508	9.5908	8.2604	7.4338
21	18.7683	17.1823	16.3444	15.4446	13.2396	11.5913	10.2829	8.8972	8.0337
22	19.7288	18.1007	17.2396	16.3140	14.0415	12.3380	10.9823	9.5425	8.6427
23	20.6902	19.0211	18.1373	17.1865	14.8480	13.0905	11.6886	10.1957	9.2604
24	21.6525	19.9432	19.0373	18.0618	15.6587	13.8484	12.4012	10.8564	9.8862
25	22.6156	20.8670	19.9393	18.9398	16.4734	14.6114	13.1197	11.5240	10.5197
30	27.4416	25.5078	24.4776	23.3641	20.5992	18.4927	16.7908	14.9535	13.7867
40	37.1340	34.8719	33.6603	32.3450	29.0505	26.5093	24.4330	22.1643	20.7065
50	46.8638	44.3133	42.9421	41.4492	37.6886	34.7643	32.3574	29.7067	27.9907
60	56.6200	53.8091	52.2938	50.6406	46.4589	43.1880	40.4817	37.4849	35.5345
70	66.3961	63.3460	61.6983	59.8978	55.3289	51.7393	48.7576	45.4417	43.2752
80	76.1879	72.9153	71.1445	69.2069	64.2778	60.3915	57.1532	53.5401	51.1719
90	85.9925	82.5111	80.6247	78.5584	73.2911	69.1260	65.6466	61.7541	59.1963
100	95.8078	92.1289	90.1332	87.9453	82.3581	77.9295	74.2219	70.0649	67.3276

Chi-Squared Values—Central Hump + Right Tails.

Values of $t_{\alpha, df}$

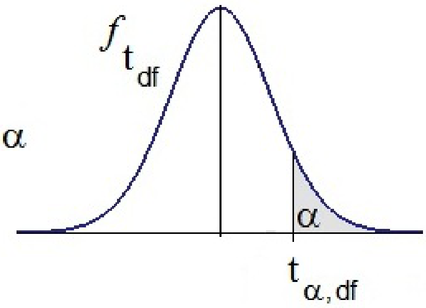
$P(\, t_{df} \geqslant t_{\alpha, df}\,) = \alpha$



df \ α	.450	.400	.350	.300	.250	.200
1	.1584	.3249	.5095	.7265	1.0000	1.3764
2	.1421	.2887	.4447	.6172	.8165	1.0607
3	.1366	.2767	.4242	.5844	.7649	.9785
4	.1338	.2707	.4142	.5686	.7407	.9410
5	.1322	.2672	.4082	.5594	.7267	.9195
6	.1311	.2648	.4043	.5534	.7176	.9057
7	.1303	.2632	.4015	.5491	.7111	.8960
8	.1297	.2619	.3995	.5459	.7064	.8889
9	.1293	.2610	.3979	.5435	.7027	.8834
10	.1289	.2602	.3966	.5415	.6998	.8791
11	.1286	.2596	.3956	.5399	.6974	.8755
12	.1283	.2590	.3947	.5386	.6955	.8726
13	.1281	.2586	.3940	.5375	.6938	.8702
14	.1280	.2582	.3933	.5366	.6924	.8681
15	.1278	.2579	.3928	.5357	.6912	.8662
16	.1277	.2576	.3923	.5350	.6901	.8647
17	.1276	.2573	.3919	.5344	.6892	.8633
18	.1274	.2571	.3915	.5338	.6884	.8620
19	.1274	.2569	.3912	.5333	.6876	.8610
20	.1273	.2567	.3909	.5329	.6870	.8600
21	.1272	.2566	.3906	.5325	.6864	.8591
22	.1271	.2564	.3904	.5321	.6858	.8583
23	.1271	.2563	.3902	.5317	.6853	.8575
24	.1270	.2562	.3900	.5314	.6848	.8569
25	.1269	.2561	.3898	.5312	.6844	.8562
26	.1269	.2560	.3896	.5309	.6840	.8557
27	.1268	.2559	.3894	.5306	.6837	.8551
28	.1268	.2558	.3893	.5304	.6834	.8546
29	.1268	.2557	.3892	.5302	.6830	.8542
30	.1267	.2556	.3890	.5300	.6828	.8538
40	.1265	.2550	.3881	.5286	.6807	.8507
50	.1263	.2547	.3875	.5278	.6794	.8489
60	.1262	.2545	.3872	.5272	.6786	.8477
70	.1261	.2543	.3869	.5268	.6780	.8468
80	.1261	.2542	.3867	.5265	.6776	.8461
90	.1260	.2541	.3866	.5263	.6772	.8456
100	.1260	.2540	.3864	.5261	.6770	.8452

Values of $t_{\alpha, df}$

$P(\, t_{df} \geqslant t_{\alpha, df} \,) = \alpha$



df \ α	.150	.100	.050	.025	.010	.005
1	1.9626	3.0777	6.3138	12.7062	31.8205	63.6567
2	1.3862	1.8856	2.9200	4.3027	6.9646	9.9248
3	1.2498	1.6377	2.3534	3.1824	4.5407	5.8409
4	1.1896	1.5332	2.1318	2.7764	3.7469	4.6041
5	1.1558	1.4759	2.0150	2.5706	3.3649	4.0321
6	1.1342	1.4398	1.9432	2.4469	3.1427	3.7074
7	1.1192	1.4149	1.8946	2.3646	2.9980	3.4995
8	1.1081	1.3968	1.8595	2.3060	2.8965	3.3554
9	1.0997	1.3830	1.8331	2.2622	2.8214	3.2498
10	1.0931	1.3722	1.8125	2.2281	2.7638	3.1693
11	1.0877	1.3634	1.7959	2.2010	2.7181	3.1058
12	1.0832	1.3562	1.7823	2.1788	2.6810	3.0545
13	1.0795	1.3502	1.7709	2.1604	2.6503	3.0123
14	1.0763	1.3450	1.7613	2.1448	2.6245	2.9768
15	1.0735	1.3406	1.7531	2.1314	2.6025	2.9467
16	1.0711	1.3368	1.7459	2.1199	2.5835	2.9208
17	1.0690	1.3334	1.7396	2.1098	2.5669	2.8982
18	1.0672	1.3304	1.7341	2.1009	2.5524	2.8784
19	1.0655	1.3277	1.7291	2.0930	2.5395	2.8609
20	1.0640	1.3253	1.7247	2.0860	2.5280	2.8453
21	1.0627	1.3232	1.7207	2.0796	2.5176	2.8314
22	1.0614	1.3212	1.7171	2.0739	2.5083	2.8188
23	1.0603	1.3195	1.7139	2.0687	2.4999	2.8073
24	1.0593	1.3178	1.7109	2.0639	2.4922	2.7969
25	1.0584	1.3163	1.7081	2.0595	2.4851	2.7874
26	1.0575	1.3150	1.7056	2.0555	2.4786	2.7787
27	1.0567	1.3137	1.7033	2.0518	2.4727	2.7707
28	1.0560	1.3125	1.7011	2.0484	2.4671	2.7633
29	1.0553	1.3114	1.6991	2.0452	2.4620	2.7564
30	1.0547	1.3104	1.6973	2.0423	2.4573	2.7500
40	1.0500	1.3031	1.6839	2.0211	2.4233	2.7045
50	1.0473	1.2987	1.6759	2.0086	2.4033	2.6778
60	1.0455	1.2958	1.6706	2.0003	2.3901	2.6603
70	1.0442	1.2938	1.6669	1.9944	2.3808	2.6479
80	1.0432	1.2922	1.6641	1.9901	2.3739	2.6387
90	1.0424	1.2910	1.6620	1.9867	2.3685	2.6316
100	1.0418	1.2901	1.6602	1.9840	2.3642	2.6259