1. Approximate the area under the graph $y = \frac{x-1}{x+1}$ over $[0, 6]$ using 3 intervals ($n = 3$) and midpoints.

(a) $\frac{7}{3}$

(b) $\frac{11}{12}$

(c) $\frac{5}{6}$

(d) $\frac{7}{6}$

(e) $\frac{5}{12}$

(f) $\frac{5}{3}$

The intervals are $[0,2], [2,4], [4,6]$, so the mid points are $1, 3, 5$, $\Delta x = \frac{6-0}{3} = 2$.

area $\approx \frac{1-1}{1+1} \times 2 + \frac{3-1}{3+1} \times 2 + \frac{5-1}{5+1} \times 2 = 0 + 1 + \frac{8}{6} = \frac{14}{6} = \frac{7}{3}$

2. Which of the following is the formula for the Riemann sum $f(x) = x^3$ over $[1, 2]$ with n-subintervals and right end points?

(a) $\sum_{i=0}^{n} \frac{1}{n}(1 + \frac{1+i}{n})^3$

(b) $\sum_{i=0}^{n} \frac{2}{n}(1 + \frac{i}{n})^3$

(c) $\sum_{i=1}^{n} \frac{1}{n}(1 + \frac{i}{n})^3$

(d) $\sum_{i=1}^{n} \frac{2}{n}(1 + \frac{1+i}{n})^3$

(e) $\sum_{i=0}^{n-1} \frac{1}{n}(1 + \frac{i}{n})^3$

(f) $\sum_{i=0}^{n-1} \frac{2}{n}(1 + \frac{1+i}{n})^3$

3. Evaluate

$$\int_1^4 \frac{3x-1}{\sqrt{x}}\,dx.$$

(a) $\frac{11}{3}$

(b) 2

(c) -2

(d) $\frac{8}{3}$

(e) $\frac{3}{2}$

(f) 12

$$\int_1^4 \frac{3x-1}{\sqrt{x}}\,dx = \int_1^4 3x^{+\frac{1}{2}} - x^{-\frac{1}{2}}\,dx = 2x^{\frac{3}{2}} - 2x^{\frac{1}{2}}\Big|_1^4 = (2\cdot 8 - 2\cdot 2)$$

$$-(2-2) = 16-4 = 12$$

4. Evaluate the integral $\int_{-1}^{3} x - 2|x-1|\, dx$.

(a) 1

(b) $-\frac{1}{2}$

(c) -4

(d) 0

(e) 2

(f) -2

On $[-1, 3]$ $x-1$ is $\begin{cases} \geq 0 & \text{if } 1 \leq x \leq 3 \\ \leq 0 & \text{if } -1 \leq x \leq 1 \end{cases}$

so $\int_{-1}^{3} x - 2|x-1|\, dx = \int_{-1}^{1} x + 2(x-1)\, dx + \int_{1}^{3} x - 2(x-1)\, dx$

$= \int_{-1}^{1} 3x - 2\, dx + \int_{1}^{3} -x + 2\, dx = \frac{3x^2}{2} - 2x \Big|_{-1}^{1} + \left(-\frac{x^2}{2} + 2x\right)\Big|_{1}^{3}$

$= \left(\frac{3}{2} - 2\right) - \left(\frac{3}{2} + 2\right) + \left(-\frac{3^2}{2} + 6\right) - \left(-\frac{1}{2} + 2\right)$

$= -4 - \frac{9}{2} + 6 + \frac{1}{2} - 2 = -4$

5. If $\int_1^5 f(x)\,dx = 2$, and $\int_3^5 f(x)\,dx = -1$, then evaluate

$$\int_1^3 2f(x) - 1 \quad dx.$$

(a) -2

(b) -1

(c) 1

(d) 2

(e) 4

(f) 8

$$\int_1^3 f(x)\,dx + \underbrace{\int_3^5 f(x)\,dx}_{-1} = \underbrace{\int_1^5 f(x)\,dx}_{2}$$

so $\int_1^3 f(x)\,dx = 3$

so $\int_1^3 2f(x) - 1 \quad dx = 2\int_1^3 f(x)\,dx - \int_1^3 1 \quad dx = 2 \times 3 - 2 = 4$

6. Evaluate $\int_{-1}^{0} xe^{x^2+1}\,dx$.

(a) $e - e^2$

(b) e^2

(c) $\frac{e}{2}$

(d) $-\frac{e}{2}$

(e) $\frac{e-1}{2}$

(f) $\frac{e-e^2}{2}$

$u = x^2+1$ $\qquad du = 2x\,dx \Rightarrow \qquad x\,dx = \frac{du}{2}$

So $\int xe^{x^2+1}\,dx = \int \frac{e^u}{2}\,du = \frac{e^u}{2} = \frac{e^{x^2+1}}{2}$

so $\int_{-1}^{0} xe^{x^2+1} = \left.\frac{e^{x^2+1}}{2}\right|_{-1}^{0} = \frac{e-e^2}{2}$

7. A particle is moving on a line with velocity function

$$v(t) = t^2 - t - 2.$$

What is the displacement of the particle from $t = 0$ to $t = 3$?

(a) $\frac{1}{3}$

(b) $\frac{3}{2}$

(c) $\frac{8}{3}$

(d) $-\frac{1}{3}$

(e) $-\frac{3}{2}$

(f) $-\frac{8}{3}$

displacement $= \int_0^3 t^2 - t - 2 \, dt = \frac{t^3}{3} - \frac{t^2}{2} - 2t \Big|_0^3$

$= \frac{27}{3} - \frac{9}{2} - 6$

$= 9 - \frac{9}{2} - 6$

$= -\frac{3}{2}$

8. What is the general indefinite integral
$$\int \frac{1}{x} + 2^x \, dx?$$

(a) $\ln x + 2^x + C$

(b) $\ln |x| + 2^x + C$

(c) $-\frac{1}{x^2} + 2^x + C$

(d) $\ln |x| + \frac{2^x}{\ln 2} + C$

(e) $\ln |x| + (\ln 2)\ 2^x + C$

(f) $\frac{1}{x^2} + (\ln 2)\ 2^x + C$

9. A particle is moving along a line with velocity function $v(t) = 3t - 6$. What is the total distance traveled from $t = 0$ to $t = 4$?

(a) 0

(b) 2

(c) 3

(d) 6

(e) 12

(f) 24

$v(t) \leq 0$ if $t \leq 2$

$v(t) \geq 0$ if $t \geq 2$

so the total distance traveled is

$$\int_0^2 -(3t-6)\,dt + \int_2^4 3t-6\,dt = -\frac{3}{2}t^2 + 6t\Big|_0^2 + \frac{3t^2}{2} - 6t\Big|_2^4$$

$$= -6 + 12 + 24 - 24 - (6-12)$$

$$= 12$$

10. What is

$$\int_{\pi/4}^{\pi/2} \frac{\cos x}{\sin^5 x}\, dx?$$

(a) $\frac{1}{4}$

(b) $-\frac{1}{2}$

(c) $\frac{\sqrt{2}}{2}$

(d) $\frac{3}{4}$

(e) $\frac{1}{3}$

(f) 1

if $u = \sin^2 x$, then $du = 2\sin x \cos x \, dx$, so $\cos x \, dx = \frac{du}{2\sin x}$

so $\int \frac{\cos x}{\sin^5 x}\, dx = \int \frac{du}{2\sin^6 x} = \int \frac{du}{2u^3} = -\frac{u^{-2}}{4} + C$

$= -\frac{\sin^{-4} x}{4}$

so $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos x}{\sin^5 x}\, dx = -\frac{1}{4\sin^4 x}\Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} = -\frac{1}{4} - \left(\frac{-1}{4\left(\frac{\sqrt{2}}{2}\right)^4}\right)$

$= -\frac{1}{4} + 1 = \frac{3}{4}$

11. By finding the maximum and minimum of $f(x) = \frac{2}{1+x^2}$ over $[0, 3]$, find a and b such that

$$a \leq \int_0^3 \frac{2}{1+x^2}\, dx \leq b$$

(a) $a = \frac{3}{5}, b = 6$

(b) $a = 3, b = 6$

(c) $a = 3, b = 5$

(d) $a = 3, b = \frac{9}{2}$

(e) $a = \frac{3}{5}, b = \frac{9}{2}$

(f) $a = \frac{1}{5}, b = 2$

$0 \leq x \leq 3$, so $0 \leq x^2 \leq 9$ so $1 \leq x^2+1 \leq 10$

so $\frac{2}{10} \leq \frac{2}{1+x^2} \leq 2$

so $\underbrace{\frac{2}{10}(3-0)}_{a} \leq \int_0^3 \frac{2}{1+x^2}\, dx \leq \underbrace{2\,(3-0)}_{b}$

12. Express the integral

$$\int_0^1 \frac{x+1}{(x^2+2x)^3}\,dx$$

as an integral in u using the substitution $u = x^2 + 2x$.

(a) $\int_0^1 \frac{1}{u^3}\,du$

(b) $\int_0^3 \frac{1}{2u^3}\,du$

(c) $\int_0^3 \frac{1}{u^3}\,du$

(d) $\int_0^2 \frac{1}{6u^3}\,du$

(e) $\int_0^1 \frac{1}{2u^2}\,du$

(f) $\int_0^1 \frac{1}{6u}\,du$

$u = x^2 + 2x \qquad du = 2x + 2 \;\; dx$, so $\frac{du}{2} = (x+1)\,dx$

$x : [0, 1]$

$x^2 + 2x : \; [0, 3]$

So the integral becomes $\int_0^3 \frac{du}{2u^3}$

13. If

$$g(x) = \int_1^x \frac{5}{t^3+3}\,dt,$$

then what is $g'(3)$?

(a) $\frac{1}{3}$

(b) $\frac{1}{6}$

(c) $\frac{1}{2}$

(d) $\frac{5}{6}$

(e) $\frac{5}{3}$

(f) $\frac{5}{2}$

By the fundamental theorem of calculus

$g'(x) = \frac{5}{x^3+3}$, so $g'(3) = \frac{5}{30} = \frac{1}{6}$

14. Find the area of the region enclosed by the curves $y = 8x^2$ and $y = 2 + 6x^2$.

(a) 2

(b) 3

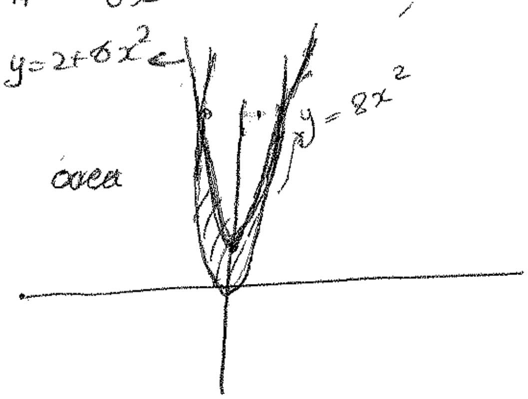
(c) $\frac{9}{4}$

(d) $\frac{4}{3}$

(e) $\frac{8}{3}$

(f) $\frac{9}{2}$

If $8x^2 = 2 + 6x^2$, then $2x^2 = 2$, so $x^2 = 1$, so $x = \pm 1$



$$\text{area} = \int_{-1}^{1} 2 + 6x^2 - 8x^2 \, dx$$

$$= \int_{-1}^{1} 2 - 2x^2 \, dx = 2x - \frac{2}{3}x^3 \Big|_{-1}^{1}$$

$$= \left(2 - \frac{2}{3}\right) - \left(-2 + \frac{2}{3}\right)$$

$$= 4 - \frac{4}{3} = \frac{8}{3}$$

in the first quadrant

15. Find the area of the region enclosed by the curve $y = \frac{1}{9}x^2$ and the two lines $y = x$ and $y = 1$.

below

(a) $\frac{3}{2}$

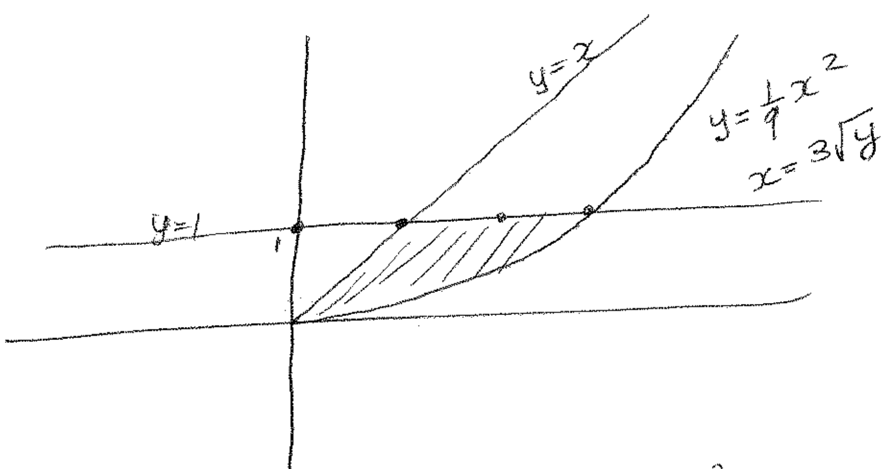
(b) $\frac{1}{2}$

(c) 1

(d) 2

(e) 3

(f) 6



$$\text{area} = \int_0^1 3\sqrt{y} - y \, dy = 2y^{\frac{3}{2}} \Big|_0^1 - \frac{y^2}{2} \Big|_0^1 = 2 - \frac{1}{2} = \frac{3}{2}$$