1. Let $N$ be a normal subgroup of a group $G$. For a subgroup $H$ in $G$, let $\phi(H) = \{hN | h \in H\} \subset G/N$.

(a) Show $\phi$ gives a 1-1 correspondence between subgroups of $G$ which contain $N$ and subgroups of $G/N$.

(b) Show that if $N \leq H_1 \leq H_2$, then $H_1 \trianglelefteq H_2$ if and only if $H_1/N \trianglelefteq H_2/N$, and in this case $H_2/H_1 \simeq (H_2/N)/(H_1/N)$.

2. (a) Show that $A_5$ is simple. (b) Find all the normal subgroups of $S_n$ when $n \geq 5$.

3. If $H \leq K \leq G$, then show that $[G : H] = [G : K][K : H]$. (Do not assume $G$ is finite.)

4. Show that if $n \geq 2$, $S_n$ is generated by the two cycles $\tau = (1\ 2)$ and $\sigma = (1\ 2\ \ldots\ n)$. (Hint: $(i\ i+1) = \sigma(i-1\ i)\sigma^{-1}$)

5. Let $G$ be the subgroup of $GL(2, \mathbb{C})$ generated by $A = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$.

(a) Show $G$ is of order 8 and is not abelian.

(b) Show that every subgroup of $G$ is normal.

6. Suppose that $G$ is a finite group, $H \leq G$ and $N \trianglelefteq G$ such that $|H|$ and $[G : N]$ are relatively prime. Show that $H \leq N$.

7. Let $G$ be a group. For $x, y$ in $G$ the *commutator* of $x$ and $y$ is $[x, y] = xyx^{-1}y^{-1}$.
Let $G'$ be the subgroup of $G$ generated by all the commutators.

- Show $[x, y]^{-1} = [y, x]$ and $G' = \{ [x_1, y_1] \cdots [x_m, y_m] \mid x_i, y_i \in G \}$.
- Show $G' \trianglelefteq G$ and $G/G'$ is abelian.
- Set $G^{(0)} = G$, and $G^{(i)} = (G^{(i-1)})'$ for $i \geq 1$. Show $G$ is solvable if and only if $G^{(m)} = \{e\}$ for some $m \geq 0$.  