1. Do the following exercises from the book:

- Exercises 6: 16, 53
- Exercises 7: 3
- Exercises 8: 18

2. Show that the greatest common divisor (gcd) of any two integers $a$ and $b$ can be written as a linear combination of $a$ and $b$

$$\text{gcd}(a, b) = xa + yb, \quad x, y \in \mathbb{Z}.$$  

(Consider the set of all positive numbers which can be written as linear combination of $a$ and $b$. Show this set is non-empty, and its smallest element is $\text{gcd}(a, b)$ by showing that it divides both $a$ and $b$ and is a multiple of any common divisor of $a$ and $b$).

3. The order of an element $g$ in a group $G$ is the smallest positive integer $m$ such that $g^m = e$. If there is no such $m$, then $g$ is said to be of infinite order.

(a) Show that for any two elements $a, b \in G$, $ab$ and $ba$ have the same order.

(b) Give an example of a group $G$ and two elements $a$ and $b$ of finite order such that $ab$ has infinite order. (Hint: you can find an example using matrices with multiplication)