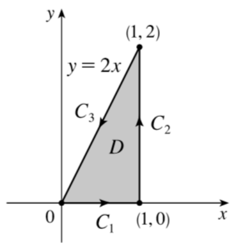
3. (a)



C_1: $x = t \;\Rightarrow\; dx = dt,\;\; y = 0 \;\Rightarrow\; dy = 0\,dt,\;\; 0 \le t \le 1.$

C_2: $x = 1 \;\Rightarrow\; dx = 0\,dt,\;\; y = t \;\Rightarrow\; dy = dt,\;\; 0 \le t \le 2.$

C_3: $x = 1 - t \;\Rightarrow\; dx = -dt,\;\; y = 2 - 2t \;\Rightarrow\; dy = -2\,dt,\;\; 0 \le t \le 1.$

Thus

$$\oint_C xy\,dx + x^2y^3\,dy = \oint_{C_1+C_2+C_3} xy\,dx + x^2y^3\,dy$$
$$= \int_0^1 0\,dt + \int_0^2 t^3\,dt + \int_0^1 \left[-(1-t)(2-2t) - 2(1-t)^2(2-2t)^3\right] dt$$
$$= 0 + \left[\tfrac{1}{4}t^4\right]_0^2 + \int_0^1 \left[-2(1-t)^2 - 16(1-t)^5\right] dt$$
$$= 4 + \left[\tfrac{2}{3}(1-t)^3 + \tfrac{8}{3}(1-t)^6\right]_0^1 = 4 + 0 - \tfrac{10}{3} = \tfrac{2}{3}$$

(b) $$\oint_C xy\,dx + x^2y^3\,dy = \iint_D \left[\frac{\partial}{\partial x}(x^2y^3) - \frac{\partial}{\partial y}(xy)\right] dA = \int_0^1 \int_0^{2x} (2xy^3 - x)\,dy\,dx$$
$$= \int_0^1 \left[\tfrac{1}{2}xy^4 - xy\right]_{y=0}^{y=2x} dx = \int_0^1 (8x^5 - 2x^2)\,dx = \tfrac{4}{3} - \tfrac{2}{3} = \tfrac{2}{3}$$

9. $$\int_C y^3\,dx - x^3\,dy = \iint_D \left[\frac{\partial}{\partial x}(-x^3) - \frac{\partial}{\partial y}(y^3)\right] dA = \iint_D (-3x^2 - 3y^2)\,dA = \int_0^{2\pi}\int_0^2 (-3r^2)\,r\,dr\,d\theta$$
$$= -3\int_0^{2\pi} d\theta \int_0^2 r^3\,dr = -3\big[\theta\big]_0^{2\pi}\left[\tfrac{1}{4}r^4\right]_0^2 = -3(2\pi)(4) = -24\pi$$

10. $$\int_C (1 - y^3)\,dx + (x^3 + e^{y^2})\,dy = \iint_D \left[\frac{\partial}{\partial x}(x^3 + e^{y^2}) - \frac{\partial}{\partial y}(1 - y^3)\right] dA = \iint_D (3x^2 + 3y^2)\,dA$$
$$= \int_0^{2\pi}\int_2^3 (3r^2)\,r\,dr\,d\theta = 3\int_0^{2\pi} d\theta \int_2^3 r^3\,dr$$
$$= 3\big[\theta\big]_0^{2\pi}\left[\tfrac{1}{4}r^4\right]_2^3 = 3(2\pi)\cdot\tfrac{1}{4}(81 - 16) = \tfrac{195}{2}\pi$$

12. $\mathbf{F}(x, y) = \langle e^{-x} + y^2, e^{-y} + x^2\rangle$ and the region D enclosed by C is given by $\{(x, y) \mid -\pi/2 \le x \le \pi/2, 0 \le y \le \cos x\}$.
C is traversed clockwise, so $-C$ gives the positive orientation.

$$\int_C \mathbf{F}\cdot d\mathbf{r} = -\int_{-C} \left(e^{-x} + y^2\right) dx + \left(e^{-y} + x^2\right) dy = -\iint_D \left[\frac{\partial}{\partial x}\left(e^{-y} + x^2\right) - \frac{\partial}{\partial y}\left(e^{-x} + y^2\right)\right] dA$$
$$= -\int_{-\pi/2}^{\pi/2}\int_0^{\cos x} (2x - 2y)\,dy\,dx = -\int_{-\pi/2}^{\pi/2} \left[2xy - y^2\right]_{y=0}^{y=\cos x} dx$$
$$= -\int_{-\pi/2}^{\pi/2} (2x\cos x - \cos^2 x)\,dx = -\int_{-\pi/2}^{\pi/2} \left[2x\cos x - \tfrac{1}{2}(1 + \cos 2x)\right] dx$$
$$= -\left[2x\sin x + 2\cos x - \tfrac{1}{2}\left(x + \tfrac{1}{2}\sin 2x\right)\right]_{-\pi/2}^{\pi/2} \qquad \text{[integrate by parts in the first term]}$$
$$= -\left(\pi - \tfrac{1}{4}\pi - \pi - \tfrac{1}{4}\pi\right) = \tfrac{1}{2}\pi$$

17. By Green's Theorem, $W = \int_C \mathbf{F}\cdot d\mathbf{r} = \int_C x(x + y)\,dx + xy^2\,dy = \iint_D (y^2 - x)\,dA$ where C is the path described in the question and D is the triangle bounded by C. So

$$W = \int_0^1\int_0^{1-x} (y^2 - x)\,dy\,dx = \int_0^1 \left[\tfrac{1}{3}y^3 - xy\right]_{y=0}^{y=1-x} dx = \int_0^1 \left(\tfrac{1}{3}(1-x)^3 - x(1-x)\right) dx$$
$$= \left[-\tfrac{1}{12}(1-x)^4 - \tfrac{1}{2}x^2 + \tfrac{1}{3}x^3\right]_0^1 = \left(-\tfrac{1}{2} + \tfrac{1}{3}\right) - \left(-\tfrac{1}{12}\right) = -\tfrac{1}{12}$$

28. P and Q have continuous partial derivatives on $\mathbb{R}^2$, so by Green's Theorem we have

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA = \iint_D (3-1)\, dA = 2 \iint_D dA = 2 \cdot A(D) = 2 \cdot 6 = 12$$