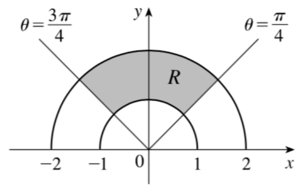
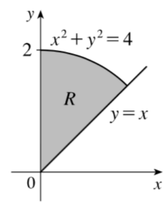
5. The integral $\int_{\pi/4}^{3\pi/4}\int_1^2 r\,dr\,d\theta$ represents the area of the region $R=\{(r,\theta)\mid 1\le r\le 2,\ \pi/4\le\theta\le 3\pi/4\}$, the top quarter portion of a ring (annulus).



$$\int_{\pi/4}^{3\pi/4}\int_1^2 r\,dr\,d\theta=\left(\int_{\pi/4}^{3\pi/4}d\theta\right)\left(\int_1^2 r\,dr\right)$$
$$=\big[\theta\big]_{\pi/4}^{3\pi/4}\left[\tfrac12 r^2\right]_1^2=\left(\tfrac{3\pi}{4}-\tfrac{\pi}{4}\right)\cdot\tfrac12(4-1)=\tfrac{\pi}{2}\cdot\tfrac32=\tfrac{3\pi}{4}$$

8. The region R is $\frac18$ of a disk, as shown in the figure, and can be described by $R=\{(r,\theta)\mid 0\le r\le 2,\ \pi/4\le\theta\le\pi/2\}$. Thus

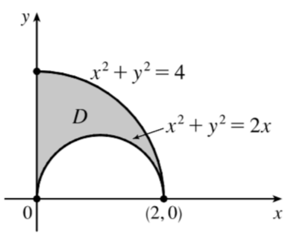


$$\iint_R(2x-y)\,dA=\int_{\pi/4}^{\pi/2}\int_0^2(2r\cos\theta-r\sin\theta)\,r\,dr\,d\theta$$
$$=\int_{\pi/4}^{\pi/2}(2\cos\theta-\sin\theta)\,d\theta\int_0^2 r^2\,dr$$
$$=\big[2\sin\theta+\cos\theta\big]_{\pi/4}^{\pi/2}\left[\tfrac13 r^3\right]_0^2$$
$$=\left(2+0-\sqrt2-\tfrac{\sqrt2}{2}\right)\left(\tfrac83\right)=\tfrac{16}{3}-4\sqrt2$$

9. $\iint_R\sin(x^2+y^2)\,dA=\int_0^{\pi/2}\int_1^3\sin(r^2)\,r\,dr\,d\theta=\int_0^{\pi/2}d\theta\int_1^3 r\sin(r^2)\,dr=\big[\theta\big]_0^{\pi/2}\left[-\tfrac12\cos(r^2)\right]_1^3$

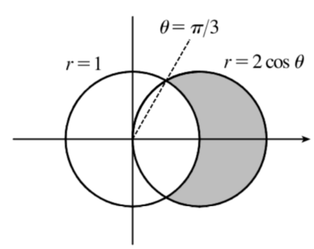
$$=\left(\tfrac{\pi}{2}\right)\left[-\tfrac12(\cos 9-\cos 1)\right]=\tfrac{\pi}{4}(\cos 1-\cos 9)$$

14.



$$\iint_D x\,dA=\iint_{\substack{x^2+y^2\le 4\\ x\ge0,\,y\ge0}}x\,dA-\iint_{\substack{(x-1)^2+y^2\le1\\ y\ge0}}x\,dA$$
$$=\int_0^{\pi/2}\int_0^2 r^2\cos\theta\,dr\,d\theta-\int_0^{\pi/2}\int_0^{2\cos\theta}r^2\cos\theta\,dr\,d\theta$$
$$=\int_0^{\pi/2}\tfrac13(8\cos\theta)\,d\theta-\int_0^{\pi/2}\tfrac13(8\cos^4\theta)\,d\theta$$
$$=\tfrac83-\tfrac{8}{12}\left[\cos^3\theta\sin\theta+\tfrac32(\theta+\sin\theta\cos\theta)\right]_0^{\pi/2}$$
$$=\tfrac83-\tfrac23\left[0+\tfrac32\left(\tfrac{\pi}{2}\right)\right]=\frac{16-3\pi}{6}$$

17. In polar coordinates the circle $(x-1)^2+y^2=1 \;\Leftrightarrow\; x^2+y^2=2x$ is $r^2=2r\cos\theta \;\Rightarrow\; r=2\cos\theta$, and the circle $x^2+y^2=1$ is $r=1$. The curves intersect in the first quadrant when $2\cos\theta=1 \;\Rightarrow\; \cos\theta=\frac12 \;\Rightarrow\; \theta=\pi/3$, so the portion of the region in the first quadrant is given by $D=\{(r,\theta)\mid 1\le r\le 2\cos\theta,\ 0\le\theta\le\pi/3\}$. By symmetry, the total area is twice the area of D:



$$2A(D)=2\iint_D dA=2\int_0^{\pi/3}\int_1^{2\cos\theta}r\,dr\,d\theta=2\int_0^{\pi/3}\left[\tfrac12 r^2\right]_{r=1}^{r=2\cos\theta}d\theta$$
$$=\int_0^{\pi/3}\left(4\cos^2\theta-1\right)d\theta=\int_0^{\pi/3}\left[4\cdot\tfrac12(1+\cos2\theta)-1\right]d\theta$$
$$=\int_0^{\pi/3}(1+2\cos2\theta)\,d\theta=\big[\theta+\sin2\theta\big]_0^{\pi/3}=\tfrac{\pi}{3}+\tfrac{\sqrt3}{2}$$

24. The paraboloid $z = 1 + 2x^2 + 2y^2$ intersects the plane $z = 7$ when $7 = 1 + 2x^2 + 2y^2$ or $x^2 + y^2 = 3$ and we are restricted to the first octant, so

$$V = \iint_{\substack{x^2+y^2 \le 3, \\ x \ge 0, y \ge 0}} \left[7 - \left(1 + 2x^2 + 2y^2\right)\right] dA = \int_0^{\pi/2} \int_0^{\sqrt{3}} \left[7 - (1 + 2r^2)\right] r\, dr\, d\theta$$

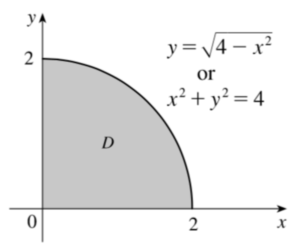
$$= \int_0^{\pi/2} d\theta \int_0^{\sqrt{3}} \left(6r - 2r^3\right) dr = \left[\,\theta\,\right]_0^{\pi/2} \left[3r^2 - \tfrac{1}{2}r^4\right]_0^{\sqrt{3}} = \tfrac{\pi}{2} \cdot \tfrac{9}{2} = \tfrac{9}{4}\pi$$

25. The cone $z = \sqrt{x^2 + y^2}$ intersects the sphere $x^2 + y^2 + z^2 = 1$ when $x^2 + y^2 + \left(\sqrt{x^2 + y^2}\right)^2 = 1$ or $x^2 + y^2 = \frac{1}{2}$. So

$$V = \iint_{x^2 + y^2 \le 1/2} \left(\sqrt{1 - x^2 - y^2} - \sqrt{x^2 + y^2}\right) dA = \int_0^{2\pi} \int_0^{1/\sqrt{2}} \left(\sqrt{1 - r^2} - r\right) r\, dr\, d\theta$$

$$= \int_0^{2\pi} d\theta \int_0^{1/\sqrt{2}} \left(r\sqrt{1 - r^2} - r^2\right) dr = \left[\,\theta\,\right]_0^{2\pi} \left[-\tfrac{1}{3}(1 - r^2)^{3/2} - \tfrac{1}{3}r^3\right]_0^{1/\sqrt{2}} = 2\pi\left(-\tfrac{1}{3}\right)\left(\tfrac{1}{\sqrt{2}} - 1\right) = \tfrac{\pi}{3}\left(2 - \sqrt{2}\right)$$

29.

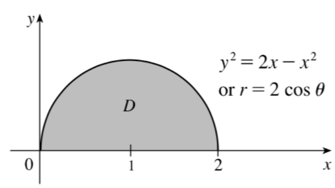


$$\int_0^2 \int_0^{\sqrt{4-x^2}} e^{-x^2-y^2}\, dy\, dx = \int_0^{\pi/2} \int_0^2 e^{-r^2} r\, dr\, d\theta$$

$$= \int_0^{\pi/2} d\theta \int_0^2 re^{-r^2} dr = \left[\,\theta\,\right]_0^{\pi/2} \left[-\tfrac{1}{2}e^{-r^2}\right]_0^2$$

$$= \tfrac{\pi}{2}\left[-\tfrac{1}{2}\left(e^{-4} - 1\right)\right] = \tfrac{\pi}{4}\left(1 - e^{-4}\right)$$

32.



$$\int_0^{\pi/2} \int_0^{2\cos\theta} r^2\, dr\, d\theta = \int_0^{\pi/2} \left[\tfrac{1}{3}r^3\right]_{r=0}^{r=2\cos\theta} d\theta = \int_0^{\pi/2} \left(\tfrac{8}{3}\cos^3\theta\right) d\theta$$

$$= \tfrac{8}{3} \int_0^{\pi/2} \left(1 - \sin^2\theta\right) \cos\theta\, d\theta$$

$$= \tfrac{8}{3}\left[\sin\theta - \tfrac{1}{3}\sin^3\theta\right]_0^{\pi/2} = \tfrac{16}{9}$$