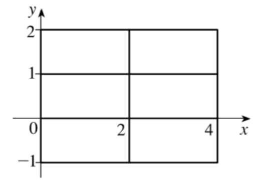
2. (a) The subrectangles are shown in the figure.



Here $\Delta A = 2$ and we estimate

$$\iint_R (1 - xy^2)\, dA \approx \sum_{i=1}^{2} \sum_{j=1}^{3} f(x_{ij}^*, y_{ij}^*)\, \Delta A$$

$$= f(2,-1)\,\Delta A + f(2,0)\,\Delta A + f(2,1)\,\Delta A + f(4,-1)\,\Delta A + f(4,0)\,\Delta A + f(4,1)\,\Delta A$$

$$= (-1)(2) + 1(2) + (-1)(2) + (-3)(2) + 1(2) + (-3)(2) = -12$$

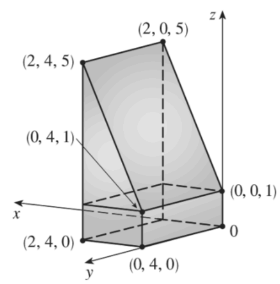
(b) $$\iint_R (1 - xy^2)\, dA \approx \sum_{i=1}^{2} \sum_{j=1}^{3} f(x_{ij}^*, y_{ij}^*)\, \Delta A$$

$$= f(0,0)\,\Delta A + f(0,1)\,\Delta A + f(0,2)\,\Delta A + f(2,0)\,\Delta A + f(2,1)\,\Delta A + f(2,2)\,\Delta A$$

$$= 1(2) + 1(2) + 1(2) + 1(2) + (-1)(2) + (-7)(2) = -8$$

10. $z = 2x + 1 \geq 0$ for $0 \leq x \leq 2$, so we can interpret the integral as the volume of the solid S that lies below the plane $z = 2x + 1$ and above the rectangle $[0,2] \times [0,4]$. We can picture S as a rectangular solid (with height 1) surmounted by a triangular cylinder; thus

$$\iint_R (2x+1)\, dA = (2)(4)(1) + \tfrac{1}{2}(2)(4)(4) = 24$$



14. $\int_0^2 y\sqrt{x+2}\, dx = \left[y \cdot \tfrac{2}{3}(x+2)^{3/2} \right]_{x=0}^{x=2} = \tfrac{2}{3}y(4)^{3/2} - \tfrac{2}{3}y(2)^{3/2} = \tfrac{16}{3}y - \tfrac{4}{3}\sqrt{2}\,y = \tfrac{4}{3}(4 - \sqrt{2})\,y$,

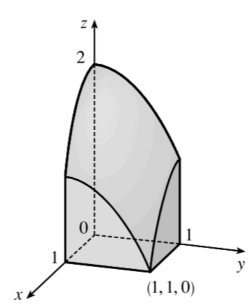
$\int_0^3 y\sqrt{x+2}\, dy = \left[\dfrac{y^2}{2}\sqrt{x+2} \right]_{y=0}^{y=3} = \tfrac{1}{2}(3)^2\sqrt{x+2} - \tfrac{1}{2}(0)^2\sqrt{x+2} = \tfrac{9}{2}\sqrt{x+2}$

20. $$\int_1^3 \int_1^5 \frac{\ln y}{xy}\, dy\, dx = \int_1^3 \frac{1}{x}\, dx \int_1^5 \frac{\ln y}{y}\, dy \qquad \text{[by Equation 11]}$$

$$= \big[\ln|x|\big]_1^3 \left[\tfrac{1}{2}(\ln y)^2\right]_1^5 \qquad [\text{substitute } u = \ln y \ \Rightarrow\ du = (1/y)\, dy]$$

$$= (\ln 3 - 0) \cdot \tfrac{1}{2}[(\ln 5)^2 - 0] = \tfrac{1}{2}(\ln 3)(\ln 5)^2$$

36. $z = 2 - x^2 - y^2 \geq 0$ for $0 \leq x \leq 1$ and $0 \leq y \leq 1$. So the solid is the region in the first octant which lies below the circular paraboloid $z = 2 - x^2 - y^2$ and above $[0,1] \times [0,1]$.



40. The solid lies under the surface $z = x^2 + xy^2$ and above the rectangle $R = [0,5] \times [-2,2]$, so its volume is

$$V = \iint_R (x^2 + xy^2)\, dA = \int_0^5 \int_{-2}^{2} (x^2 + xy^2)\, dy\, dx = \int_0^5 \left[x^2 y + \tfrac{1}{3}xy^3 \right]_{y=-2}^{y=2} dx$$

$$= \int_0^5 \left[(2x^2 + \tfrac{8}{3}x) - (-2x^2 - \tfrac{8}{3}x) \right] dx = \int_0^5 (4x^2 + \tfrac{16}{3}x)\, dx$$

$$= \left[\tfrac{4}{3}x^3 + \tfrac{8}{3}x^2 \right]_0^5 = \tfrac{500}{3} + \tfrac{200}{3} - 0 = \tfrac{700}{3}$$

42. The cylinder intersects the xy-plane along the line $x = 4$, so in the first octant, the solid lies below the surface $z = 16 - x^2$ and above the rectangle $R = [0, 4] \times [0, 5]$ in the xy-plane.

$$V = \int_0^5 \int_0^4 (16 - x^2)\, dx\, dy = \int_0^4 (16 - x^2)\, dx \int_0^5 dy$$

$$= \left[16x - \tfrac{1}{3}x^3\right]_0^4 \left[\, y\,\right]_0^5 = (64 - \tfrac{64}{3} - 0)(5 - 0) = \tfrac{640}{3}$$