4. $z = f(x,y) = x/y^2 = xy^{-2} \quad\Rightarrow\quad f_x(x,y) = 1/y^2,\;\; f_y(x,y) = -2xy^{-3} = -2x/y^3,$ so $f_x(-4,2) = \frac{1}{4}$ and $f_y(-4,2) = 1$. Thus an equation of the tangent plane is $z - (-1) = f_x(-4,2)\,[x - (-4)] + f_y(-4,2)(y-2) \quad\Rightarrow$ $z + 1 = \frac{1}{4}(x+4) + 1(y-2)$ or $z = \frac{1}{4}x + y - 2$.

5. $z = f(x,y) = x\sin(x+y) \quad\Rightarrow\quad f_x(x,y) = x\cdot\cos(x+y) + \sin(x+y)\cdot 1 = x\cos(x+y) + \sin(x+y),$ $f_y(x,y) = x\cos(x+y)$, so $f_x(-1,1) = (-1)\cos 0 + \sin 0 = -1,\;\; f_y(-1,1) = (-1)\cos 0 = -1$ and an equation of the tangent plane is $z - 0 = (-1)(x+1) + (-1)(y-1)$ or $x + y + z = 0$.

17. Let $f(x,y) = e^x\cos(xy)$. Then $f_x(x,y) = e^x[-\sin(xy)](y) + e^x\cos(xy) = e^x[\cos(xy) - y\sin(xy)]$ and $f_y(x,y) = e^x[-\sin(xy)](x) = -xe^x\sin(xy)$. Both f_x and f_y are continuous functions, so by Theorem 8, f is differentiable at $(0,0)$. We have $f_x(0,0) = e^0(\cos 0 - 0) = 1,\;\; f_y(0,0) = 0$ and the linear approximation of f at $(0,0)$ is $f(x,y) \approx f(0,0) + f_x(0,0)(x-0) + f_y(0,0)(y-0) = 1 + 1x + 0y = x + 1$.

18. Let $f(x,y) = \dfrac{y-1}{x+1}$. Then $f_x(x,y) = (y-1)(-1)(x+1)^{-2} = \dfrac{1-y}{(x+1)^2}$ and $f_y(x,y) = \dfrac{1}{x+1}$. Both f_x and f_y are continuous functions for $x \neq -1$, so by Theorem 8, f is differentiable at $(0,0)$. We have $f_x(0,0) = 1,\;\; f_y(0,0) = 1$ and the linear approximation of f at $(0,0)$ is $f(x,y) \approx f(0,0) + f_x(0,0)(x-0) + f_y(0,0)(y-0) = -1 + 1x + 1y = x + y - 1$.

19. We can estimate $f(2.2, 4.9)$ using a linear approximation of f at $(2,5)$, given by $f(x,y) \approx f(2,5) + f_x(2,5)(x-2) + f_y(2,5)(y-5) = 6 + 1(x-2) + (-1)(y-5) = x - y + 9$. Thus $f(2.2, 4.9) \approx 2.2 - 4.9 + 9 = 6.3$.

21. $f(x,y,z) = \sqrt{x^2+y^2+z^2} \quad\Rightarrow\quad f_x(x,y,z) = \dfrac{x}{\sqrt{x^2+y^2+z^2}},\;\; f_y(x,y,z) = \dfrac{y}{\sqrt{x^2+y^2+z^2}}$, and $f_z(x,y,z) = \dfrac{z}{\sqrt{x^2+y^2+z^2}}$, so $f_x(3,2,6) = \frac{3}{7},\;\; f_y(3,2,6) = \frac{2}{7},\;\; f_z(3,2,6) = \frac{6}{7}$. Then the linear approximation of f at $(3,2,6)$ is given by

$$\begin{aligned} f(x,y,z) &\approx f(3,2,6) + f_x(3,2,6)(x-3) + f_y(3,2,6)(y-2) + f_z(3,2,6)(z-6) \\ &= 7 + \tfrac{3}{7}(x-3) + \tfrac{2}{7}(y-2) + \tfrac{6}{7}(z-6) = \tfrac{3}{7}x + \tfrac{2}{7}y + \tfrac{6}{7}z \end{aligned}$$

Thus $\sqrt{(3.02)^2 + (1.97)^2 + (5.99)^2} = f(3.02, 1.97, 5.99) \approx \frac{3}{7}(3.02) + \frac{2}{7}(1.97) + \frac{6}{7}(5.99) \approx 6.9914$.

25. $z = e^{-2x}\cos 2\pi t \quad\Rightarrow$

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial t}dt = e^{-2x}(-2)\cos 2\pi t\,dx + e^{-2x}(-\sin 2\pi t)(2\pi)\,dt = -2e^{-2x}\cos 2\pi t\,dx - 2\pi e^{-2x}\sin 2\pi t\,dt$$

26. $u = \sqrt{x^2+3y^2} = (x^2+3y^2)^{1/2} \quad\Rightarrow$

$$du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy = \tfrac{1}{2}(x^2+3y^2)^{-1/2}(2x)\,dx + \tfrac{1}{2}(x^2+3y^2)^{-1/2}(6y)\,dy = \frac{x}{\sqrt{x^2+3y^2}}\,dx + \frac{3y}{\sqrt{x^2+3y^2}}\,dy$$

41. (a) $B(m,h) = m/h^2 \quad \Rightarrow \quad B_m(m,h) = 1/h^2$ and $B_h(m,h) = -2m/h^3$. Since $h > 0$, both B_m and B_h are continuous functions, so B is differentiable at $(23, 1.10)$. We have $B(23, 1.10) = 23/(1.10)^2 \approx 19.01$, $B_m(23, 1.10) = 1/(1.10)^2 \approx 0.8264$, and $B_h(23, 1.10) = -2(23)/(1.10)^3 \approx -34.56$, so the linear approximation of B at $(23, 1.10)$ is $B(m,h) \approx B(23, 1.10) + B_m(23, 1.10)(m-23) + B_h(23, 1.10)(h-1.10) \approx 19.01 + 0.8264(m-23) - 34.56(h-1.10)$ or $B(m,h) \approx 0.8264m - 34.56h + 38.02$.

(b) From part (a), for values near $m = 23$ and $h = 1.10$, $B(m,h) \approx 0.8264m - 34.56h + 38.02$. If m increases by 1 kg to 24 kg and h increases by 0.03 m to 1.13 m, we estimate the BMI to be $B(24, 1.13) \approx 0.8264(24) - 34.56(1.13) + 38.02 \approx 18.801$. This is very close to the actual computed BMI: $B(24, 1.13) = 24/(1.13)^2 \approx 18.796$.