10. $f(x,y) = (5y^4 \cos^2 x)/(x^4 + y^4)$. First approach $(0,0)$ along the x-axis. Then $f(x,0) = 0/x^4 = 0$ for $x \neq 0$, so $f(x,y) \to 0$. Next approach $(0,0)$ along the y-axis. For $y \neq 0$, $f(0,y) = 5y^4/y^4 = 5$, so $f(x,y) \to 5$. Since f has two different limits along two different lines, the limit does not exist.

11. $f(x,y) = (y^2 \sin^2 x)/(x^4 + y^4)$. On the x-axis, $f(x,0) = 0$ for $x \neq 0$, so $f(x,y) \to 0$ as $(x,y) \to (0,0)$ along the x-axis. Approaching $(0,0)$ along the line $y = x$, $f(x,x) = \dfrac{x^2 \sin^2 x}{x^4 + x^4} = \dfrac{\sin^2 x}{2x^2} = \dfrac{1}{2}\left(\dfrac{\sin x}{x}\right)^2$ for $x \neq 0$ and $\lim\limits_{x \to 0} \dfrac{\sin x}{x} = 1$, so $f(x,y) \to \frac{1}{2}$. Since f has two different limits along two different lines, the limit does not exist.

12. $f(x,y) = \dfrac{xy - y}{(x-1)^2 + y^2}$. On the x-axis, $f(x,0) = 0/(x-1)^2 = 0$ for $x \neq 1$, so $f(x,y) \to 0$ as $(x,y) \to (1,0)$ along the x-axis. Approaching $(1,0)$ along the line $y = x - 1$, $f(x,x-1) = \dfrac{x(x-1) - (x-1)}{(x-1)^2 + (x-1)^2} = \dfrac{(x-1)^2}{2(x-1)^2} = \dfrac{1}{2}$ for $x \neq 1$, so $f(x,y) \to \frac{1}{2}$ along this line. Thus the limit does not exist.

16. We can use the Squeeze Theorem to show that $\lim\limits_{(x,y)\to(0,0)} \dfrac{xy^4}{x^4 + y^4} = 0$:

$0 \leq \dfrac{|x|\, y^4}{x^4 + y^4} \leq |x|$ since $0 \leq \dfrac{y^4}{x^4 + y^4} \leq 1$, and $|x| \to 0$ as $(x,y) \to (0,0)$, so $\dfrac{|x|\, y^4}{x^4 + y^4} \to 0 \quad \Rightarrow \quad \dfrac{xy^4}{x^4 + y^4} \to 0$ as $(x,y) \to (0,0)$.

17.

$$\lim_{(x,y)\to(0,0)} \frac{x^2 + y^2}{\sqrt{x^2 + y^2 + 1} - 1} = \lim_{(x,y)\to(0,0)} \frac{x^2 + y^2}{\sqrt{x^2 + y^2 + 1} - 1} \cdot \frac{\sqrt{x^2 + y^2 + 1} + 1}{\sqrt{x^2 + y^2 + 1} + 1}$$

$$= \lim_{(x,y)\to(0,0)} \frac{(x^2 + y^2)\left(\sqrt{x^2 + y^2 + 1} + 1\right)}{x^2 + y^2} = \lim_{(x,y)\to(0,0)} \left(\sqrt{x^2 + y^2 + 1} + 1\right) = 2$$

25. $h(x,y) = g(f(x,y)) = (2x + 3y - 6)^2 + \sqrt{2x + 3y - 6}$. Since f is a polynomial, it is continuous on $\mathbb{R}^2$ and g is continuous on its domain $\{t \mid t \geq 0\}$. Thus h is continuous on its domain $\{(x,y) \mid 2x + 3y - 6 \geq 0\} = \left\{(x,y) \mid y \geq -\frac{2}{3}x + 2\right\}$, which consists of all points on or above the line $y = -\frac{2}{3}x + 2$.

34. $G(x,y) = \ln(1 + x - y) = g(f(x,y))$ where $f(x,y) = 1 + x - y$, a polynomial and hence continuous on $\mathbb{R}^2$, and $g(t) = \ln t$, continuous on its domain $\{t \mid t > 0\}$. Thus G is continuous on its domain $\{(x,y) \mid 1 + x - y > 0\} = \{(x,y) \mid y < x + 1\}$, the region in $\mathbb{R}^2$ below the line $y = x + 1$.

35. $f(x,y,z) = h(g(x,y,z))$ where $g(x,y,z) = x^2 + y^2 + z^2$, a polynomial that is continuous everywhere, and $h(t) = \arcsin t$, continuous on $[-1,1]$. Thus f is continuous on its domain $\left\{(x,y,z) \mid -1 \leq x^2 + y^2 + z^2 \leq 1\right\} = \left\{(x,y,z) \mid x^2 + y^2 + z^2 \leq 1\right\}$, so f is continuous on the unit ball.

36. $\sqrt{y - x^2}$ is continuous on its domain $\left\{(x,y) \mid y - x^2 \geq 0\right\} = \left\{(x,y) \mid y \geq x^2\right\}$ and $\ln z$ is continuous on its domain $\{z \mid z > 0\}$, so the product $f(x,y,z) = \sqrt{y - x^2}\, \ln z$ is continuous for $y \geq x^2$ and $z > 0$, that is, $\left\{(x,y,z) \mid y \geq x^2,\, z > 0\right\}$.

37. $f(x,y)=\begin{cases}\dfrac{x^2y^3}{2x^2+y^2} & \text{if } (x,y)\neq(0,0)\\ 1 & \text{if } (x,y)=(0,0)\end{cases}$ The first piece of f is a rational function defined everywhere except at the origin, so f is continuous on $\mathbb{R}^2$ except possibly at the origin. Since $x^2 \leq 2x^2+y^2$, we have $\left|x^2y^3/(2x^2+y^2)\right| \leq \left|y^3\right|$. We know that $\left|y^3\right| \to 0$ as $(x,y)\to(0,0)$. So, by the Squeeze Theorem, $\lim\limits_{(x,y)\to(0,0)} f(x,y) = \lim\limits_{(x,y)\to(0,0)} \dfrac{x^2y^3}{2x^2+y^2} = 0$. But $f(0,0)=1$, so f is discontinuous at $(0,0)$. Therefore, f is continuous on the set $\{(x,y) \mid (x,y)\neq(0,0)\}$.

40. $$\lim_{(x,y)\to(0,0)}(x^2+y^2)\ln(x^2+y^2) = \lim_{r\to0^+} r^2\ln r^2 = \lim_{r\to0^+}\frac{\ln r^2}{1/r^2} = \lim_{r\to0^+}\frac{(1/r^2)(2r)}{-2/r^3} \quad \text{[using l'Hospital's Rule]}$$
$$= \lim_{r\to0^+}(-r^2) = 0$$

41. $$\lim_{(x,y)\to(0,0)}\frac{e^{-x^2-y^2}-1}{x^2+y^2} = \lim_{r\to0^+}\frac{e^{-r^2}-1}{r^2} = \lim_{r\to0^+}\frac{e^{-r^2}(-2r)}{2r} \quad \text{[using l'Hospital's Rule]}$$
$$= \lim_{r\to0^+} -e^{-r^2} = -e^0 = -1$$