Using the TI-83 for Simpson's Rule

In order to use this information, you need to be familiar with creating and storing lists on the TI-83. This information is illustrated in the earlier documents *Approximating Areas on the TI-83* and *The Midpoint Rule on the TI-83* (see the Daily Assignments section of the web syllabus).

You also need to know an additional fact about handling lists: we have two lists of the same length, the TI-83 will "multiply" them "term-by-term" to create a new list. For example, if L1 $=$ {2,4,6} and L2 $=$ {3,2,7}, then on the TI-83

L1*L2 $=$ {2*3, 4*2, 6*7} $=$ {6,8,42}

Recall Simpson's Rule: to approximate $\int_a^b f(x)\,dx$, subdivide $[a,b]$ into n equal subintervals, each of length h (or Δx) $= \frac{b-a}{n}$. For Simpson's Rule, n must be an <u>even</u> positive integer. The endpoints of the subintervals are $x_0 = a,\ x_1, x_2, ..., x_{n-1}, x_n = b$.

Simpson's approximation to $\int_a^b f(x)\,dx$ is then given by the formula

$$S_n = \frac{h}{3}(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3)\ +\ 2f(x_4) +\ ... + 4f(x_{n-1}) + f(x_n))$$

For a small n, it's easy to compute this. For a larger n, computing it one term at a time gets tedious, and it's harder to see how to do it efficiently on the TI-83 (unlike, say, for the Midpoint Rule) because of the pattern of "switching coefficients" $\{1,4,2,4,2,4,...,2,4,1\}$. Here is a series of steps that do the job, followed by a detailed discussion of why they work.

Enter the function Y1=1/(1+x^2) in the "Y=" window.
Store the values of a, b, h ($= \Delta x$) in the TI-83

a STO A
b STO B
n STO N
(B $-$ A)/N STO H

Then

seq(A+I*H, I, 1, N $-$ 1) STO L1
seq(3+($-$ 1)^(I $-$ 1), I, 1, N $-$ 1) STO L2
(H/3)*(Y1(A) + sum(Y1(L1)*L2) + Y1(B)) (Gives the value of S_n)

Here's a step by step discussion of what's happening in a specific case. We compute S_8 for $\int_0^3 \frac{1}{1+x^2}\,dx$.

Enter Y1(X)=1/(1+x^2) in the "Y=" window.

0 STO A	Stores 0 in location A
3 STO B	Stores 3 in location B
8 STO N	Stores n in location N
(B $-$ A)/N STO H	Stores $h = \Delta x = \frac{3}{8}$ in location H

The subdivisions of $[0,3]$ and their endpoints look like

$0 \quad \frac{3}{8} \quad \frac{6}{8} \quad \frac{9}{8} \quad \frac{12}{8} \quad \frac{15}{8} \quad \frac{18}{8} \quad \frac{21}{8} \quad \frac{24}{8}$

A $=$ 0 $\qquad$ 3 $=$ B

We want to find

$$S_8 = \frac{H}{3}(\text{ Y1(A)} + 4\text{*Y1}(\tfrac{3}{8}) + 2\text{*Y1}(\tfrac{6}{8}) + 4\text{*Y1}(\tfrac{9}{8}) \; + \; 2\text{*Y1}(\tfrac{12}{8}) + 4*\text{Y1}(\tfrac{15}{8}) + 2\text{*Y1}(\tfrac{18}{8}) + 4\text{*Y1}(\tfrac{21}{8}) + \text{Y1(B)})$$

We need to evaluate the function Y1 at the 9 points in list $\{\text{A},\frac{3}{8}, \frac{6}{8}, \frac{9}{8}, \frac{12}{8}, \frac{15}{8}, \frac{18}{8}, \frac{21}{8}, \text{B}\}$
No matter how large n is, A and B are "special": those are the only points in the sum for $S_n$where the coefficient in the formula is one. So we treat them as "special" and make a list of the other N − 1 (=7) points and store in the list variable L1:

seq(A+I*H, I, 1, N − 1) STO L1 Creates the list
$\text{L1} = \{\frac{3}{8}, \frac{6}{8}, \frac{9}{8}, \frac{12}{8}, \frac{15}{8}, \frac{18}{8}, \frac{21}{8}\}$
(*Be sure you think about why*!)

In the formula for S_8, the pattern of coefficients associated with these endpoints looks like

$$\{\tfrac{3}{8}, \ \tfrac{6}{8}, \ \tfrac{9}{8}, \tfrac{12}{8}, \ \tfrac{15}{8}, \ \tfrac{18}{8}, \ \tfrac{21}{8}\}$$
$$\{4, \ 2, \ 4, 2, \ 4, \ 2, \ 4\}$$

We create a list of these coefficients and store it in the list variable L2:

seq(3+(− 1)^(I − 1), I, 1, N − 1) STO L2 Creates the list L2 = {4,2,4,2,4,2,4}
(*Be sure you think about why*!)

The commands:

Y1(L1) Creates
$\{\text{Y1}(\frac{3}{8}), \text{Y1}(\frac{6}{8}), \text{Y1}(\frac{9}{8}), \ldots, \text{Y1}(\frac{21}{8})\}$

Y1(L1)*L2 Creates
$\{4\text{*Y1}(\frac{3}{8}), 2\text{*Y1}(\frac{6}{8}), 4\text{*Y1}(\frac{9}{8}), \ldots, 4\text{*Y1}(\frac{21}{8})\}$

sum(Y1(L1)*L2) Creates
$4\text{*Y1}(\frac{3}{8}) + 2\text{*Y1}(\frac{6}{8}) + 4\text{*Y1}(\frac{9}{8}) + \ldots + 4\text{*Y1}(\frac{21}{8})$

Therefore we can get S_8 from the command

(H/3)*(Y1(A) + sum(Y1(L1)*L2) + Y1(B))

If you do these steps on the TI-83, it gives the result 1.24879899. In this example, we happen to know that the exact value of $\int_0^3 \frac{1}{1+x^2}\,dx = \arctan 3$. So, just out of curiosity, we can see how good or bad the approximation is:

Error = (Exact value) − (Approximation) = $\arctan 3 - 1.248799 \approx 0.000246782$.

Of course, if you follow the steps above, it's no more work to compute, say, S_{100}.