

1. If $P(1, 1, 0)$ and $Q(-1, 0, 3)$ are two points, which of the following is a vector of length 2 in the direction of \vec{PQ} ?

(a) $\left\langle \frac{-4}{\sqrt{14}}, \frac{-2}{\sqrt{14}}, \frac{6}{\sqrt{14}} \right\rangle$

(b) $\left\langle \frac{-2}{\sqrt{14}}, \frac{-1}{\sqrt{14}}, \frac{3}{\sqrt{14}} \right\rangle$

(c) $\left\langle \frac{2}{\sqrt{14}}, \frac{1}{\sqrt{14}}, \frac{-3}{\sqrt{14}} \right\rangle$

(d) $\left\langle \frac{4}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{-6}{\sqrt{14}} \right\rangle$

(e) $\left\langle \frac{-2}{\sqrt{7}}, \frac{-1}{\sqrt{7}}, \frac{3}{\sqrt{7}} \right\rangle$

(f) $\left\langle \frac{2}{\sqrt{7}}, \frac{1}{\sqrt{7}}, \frac{-3}{\sqrt{7}} \right\rangle$

$$\vec{PQ} = \langle -2, -1, 3 \rangle \quad |\vec{PQ}| = \sqrt{14}$$

$$\frac{\vec{PQ}}{|\vec{PQ}|} = \left\langle \frac{-4}{\sqrt{14}}, \frac{-2}{\sqrt{14}}, \frac{6}{\sqrt{14}} \right\rangle$$

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2. Which of the following vectors is perpendicular to $\langle 2, 1, -3 \rangle$?

(a) $\langle 3, 1, 2 \rangle$

(b) $\langle 1, -1, 1 \rangle$

(c) $\langle 2, -1, 1 \rangle$

(d) $\langle 2, 0, 2 \rangle$

(e) $\langle 1, 5, -1 \rangle$

(f) $\langle 1, 0, -1 \rangle$

3. For which values of a the two vectors $\vec{u} = \langle 1, a, 2 \rangle$ and $\vec{v} = \langle a, 4, 4 \rangle$ are parallel?

(a) 0

(b) $\frac{1}{4}$

(c) $\frac{1}{2}$

(d) 1

(e) 2

(f) 4

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4. If $\vec{u} = \langle 2, 1, -3 \rangle$ and $\vec{v} = \langle 1, 0, 1 \rangle$, what is $\vec{u} \times \vec{v}$?

(a) $\langle 1, -5, 2 \rangle$

(b) $\langle -1, -5, 2 \rangle$

(c) $\langle -1, -5, -2 \rangle$

(d) $\langle 1, -1, 2 \rangle$

(e) $\langle 1, -5, -1 \rangle$

(f) $\langle -1, 5, -1 \rangle$

5. If \vec{a} and \vec{b} are two vectors such that \vec{b} is of length 2 and $\vec{a} \cdot \vec{b} = 1$, what is $3\vec{b} \cdot (4\vec{a} - \vec{b})$?

(a) 0

(b) 2

(c) -6

(d) 6

(e) 10

(f) 8

$$\begin{aligned}
 3\vec{b} \cdot (4\vec{a} - \vec{b}) &= 12(\vec{b} \cdot \vec{a}) - 3(\vec{b} \cdot \vec{b}) \\
 &= 12(\vec{a} \cdot \vec{b}) - 3|\vec{b}|^2 \\
 &= 12 - 12 \\
 &= 0
 \end{aligned}$$

6. If $\vec{a} \times \vec{b} = \langle 1, 1, -1 \rangle$ and $\vec{a} \cdot \vec{b} = \sqrt{3}$, what is the angle between \vec{a} and \vec{b} ?

(a) $\frac{\pi}{6}$

(b) $\frac{\pi}{4}$

(c) $\cos^{-1}\left(\frac{2}{3}\right)$

(d) $\frac{\pi}{3}$

(e) $\cos^{-1}\left(\frac{\sqrt{3}}{3}\right)$

(f) $\cos^{-1}\left(\frac{3}{2}\right)$

$$|\vec{a} \times \vec{b}| = \sqrt{3} \quad \text{so} \quad |\vec{a}| |\vec{b}| \sin \theta = \sqrt{3}$$

$$\vec{a} \cdot \vec{b} = \sqrt{3} \quad \text{so} \quad |\vec{a}| |\vec{b}| \cos \theta = \sqrt{3}$$

Therefore $\sin \theta = \cos \theta$, so $\theta = \frac{\pi}{4}$

7. What is the center and the radius of the sphere with equation

$$x^2 + y^2 + z^2 - 2x + 4z + \frac{11}{4} = 0?$$

(a) $(1, 0, 2), \frac{3}{2}$

(b) $(-1, 0, 1), \frac{5}{4}$

(c) $(-1, 0, 2), \frac{9}{4}$

(d) $(1, 0, -2), \frac{3}{2}$

(e) $(1, 0, 1), \frac{5}{4}$

(f) $(1, 0, -2), \frac{9}{4}$

$$x^2 + y^2 + z^2 - 2x + 4z + \frac{11}{4} = (x-1)^2 + y^2 + (z+2)^2 - 1 - 4 + \frac{11}{4}$$

so the equation becomes :

$$(x-1)^2 + y^2 + (z+2)^2 = 5 - \frac{11}{4} = \frac{9}{4}$$

8. If $\vec{a} = \vec{i} + 2\vec{j}$ and $\vec{b} = \vec{i} - 2\vec{j} + 2\vec{k}$, then what is the vector projection of \vec{a} onto \vec{b} ?

(a) $\left\langle \frac{-1}{3}, \frac{2}{3}, \frac{-2}{3} \right\rangle$

(b) $\left\langle \frac{1}{3}, \frac{-2}{3}, \frac{2}{3} \right\rangle$

(c) $\left\langle \frac{-1}{9}, \frac{2}{9}, \frac{-2}{9} \right\rangle$

(d) $\left\langle \frac{1}{9}, \frac{-2}{9}, \frac{2}{9} \right\rangle$

(e) $\left\langle -1, 2, -2 \right\rangle$

(f) $\left\langle 1, -2, 2 \right\rangle$

$$\text{Proj}_{\vec{b}} \vec{a} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2} \vec{b} = \frac{-3}{9} \vec{b} = -\frac{1}{3} \left\langle 1, -2, 2 \right\rangle$$

$$\vec{a} \cdot \vec{b} = \left\langle 1, 2, 0 \right\rangle \cdot \left\langle 1, -2, 2 \right\rangle = -3$$

$$|\vec{b}|^2 = 9$$

9. What is the acute angle between the two planes $x + z = 8$ and $x + y = -1$.

(a) $\frac{\pi}{2}$

(b) $\frac{\pi}{3}$

(c) $\frac{\pi}{6}$

(d) $\frac{\pi}{4}$

(e) $\frac{3\pi}{4}$

(f) the two planes are parallel.

$$\Gamma_1 : x + z = 8 \quad \vec{n}_1 = \langle 1, 0, 1 \rangle$$

$$\Gamma_2 : x + y = -1 \quad \vec{n}_2 = \langle 1, 1, 0 \rangle$$

$$\frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1| |\vec{n}_2|} = \frac{1}{\sqrt{2} \sqrt{2}} = \frac{1}{2}, \text{ so cosine of the angle}$$

is $\frac{1}{2}$

10. Find the distance between the two parallel planes $x - 2y - z = 1$ and $x - 2y - z = 4$.

(a) $\sqrt{\frac{3}{2}}$

(b) $\sqrt{2}$

(c) 2

(d) $\sqrt{3}$

(e) 3

(f) $\frac{3}{2}$

$$\begin{aligned} \Gamma_1 : x - 2y - z &= 1 & \text{a point on } \Gamma_1: P(1, 0, 0) & \vec{n}_1 = \langle 1, -2, -1 \rangle \\ \Gamma_2 : x - 2y - z &= 4 & \dots & \dots \Gamma_2: Q(4, 0, 0) \end{aligned}$$

$$\text{distance} = \frac{|\vec{n}_1 \cdot \vec{PQ}|}{|\vec{n}_1|} = \frac{| \langle 1, -2, -1 \rangle \cdot \langle 3, 0, 0 \rangle |}{\sqrt{6}} = \frac{3}{\sqrt{6}} = \sqrt{\frac{3}{2}}$$

11. What is the area of the triangle with vertices $P(0, 1, 2)$, $Q(-1, 2, 2)$ and $R(4, -1, 0)$?

(a) $\sqrt{2}$

(b) 2

(c) 4

(d) $\sqrt{8}$

(e) 8

(f) $\sqrt{3}$

$$\vec{PQ} = \langle -1, 1, 0 \rangle \quad \vec{PR} = \langle 4, -2, -2 \rangle$$

$$\vec{PQ} \times \vec{PR} = \langle -2, -2, -2 \rangle$$

$$|\vec{PQ} \times \vec{PR}| = \sqrt{12}$$

$$\text{area of the triangle} = \frac{\sqrt{12}}{2} = \sqrt{3}$$

12. What is the equation of the plane that contains the point $(1, -1, 1)$ and contains the line given by the parametric equation

$$x = t, \quad y = 2t, \quad z = 3t?$$

(a) $x + y - z = 0$

(b) $5x - 2y - 3z = 6$

(c) $5x + 2y - 3z = 6$

(d) $\textcircled{5} 5x + 2y - 3z = 0$

(e) $x - 2y + 3z = 6$

(f) $5x - 2y + 3z = 0$

(1) $P(1, -1, 1)$ is on the plane.

(2) If $t=0$ $Q(0, 0, 0)$ is also on the plane.

(3) If $t=1$ $R(1, 2, 3)$ is also on the plane.

$$\vec{PQ} = \langle -1, 1, -1 \rangle \quad \vec{PR} = \langle 0, 3, 2 \rangle$$

$$\vec{PQ} \times \vec{PR} = \langle 5, 2, -3 \rangle$$

So the equation is $5x + 2y - 3z = 5 \cdot 1 + 2 \cdot (-1) + (-3) \cdot 1 = 0$

13. What is a parametric equation of the line through $(1, 0, -1)$ and parallel to the line with the following symmetric equation?

$$x - 1 = \frac{y + 1}{2} = z$$

(a) $x = -1 + t, y = 2 + t, z = -1 + t$

(b) $x = 1 + t, y = 2 + t, z = -1 + t$

(c) $x = 1 + t, y = 2 + t, z = -1 - t$

(d) $x = 1 + t, y = 2t, z = 1 - t$

(e) $x = -1 + t, y = 2t, z = -1 + t$

(f) $x = 1 + t, y = 2t, z = -1 + t$

The vector $\langle 1, 2, 1 \rangle$ is parallel to the line

$$x - 1 = \frac{y + 1}{2} = z$$

So the equation of the line is

$$x = 1 + t \quad y = 2t \quad z = -1 + t$$

14. What is the volume of the parallelepiped formed by the three vectors $\langle 2, 1, 1 \rangle$, $\langle -2, 0, 3 \rangle$, and $\langle 0, 1, 1 \rangle$?

(a) 1

(b) 2

(c) 3

(d) 4

(e) 5

(f) 6

$$\langle 2, 1, 1 \rangle \times \langle -2, 0, 3 \rangle = \langle 3, -8, 2 \rangle$$

$$\langle 3, -8, 2 \rangle \cdot \langle 0, 1, 1 \rangle = -8 + 2 = -6$$

15. What is the parametric equation of the line of intersection of the two planes given by equations $x+y+z=0$ and $2x-y+3z=3$?

(a) $x = 1 + 4t, \quad y = -1 - t, \quad z = -3t$

(b) $x = 1 + 2t, \quad y = -1 - t, \quad z = -3t$

(c) $x = 1 + 2t, \quad y = -1, \quad z = -3t$

(d) $x = 1 + 4t, \quad y = -1 - t, \quad z = 3t$

(e) $x = 1 + 2t, \quad y = -1, \quad z = -3t$

(f) $x = 1 + 4t, \quad y = -1, \quad z = -3t$

$$\langle 1, 1, 1 \rangle \times \langle 2, -1, 3 \rangle = \langle 4, -1, -3 \rangle$$

16. We are give three lines:

$$L_1 : \quad x = 2t, \quad y = 1 + t, \quad z = -3 + t$$

$$L_2 : \quad x = 3 + t, \quad y = 3t, \quad z = -3 + 2t$$

$$L_3 : \quad x = 2 - 2t, \quad y = 3 - 6t, \quad z = -1 - 4t$$

Which of the following statements is true?

- (a) The three lines are parallel.
- (b) L_1 and L_2 are parallel, and L_2 and L_3 are skew.
- (c) L_1 and L_2 are parallel, and L_2 and L_3 intersect.
- (d) L_1 and L_2 intersect, and L_2 and L_3 are parallel.
- (e) L_1 and L_2 are skew, and L_2 and L_3 are parallel.
- (f) L_1 and L_2 intersect, and L_2 and L_3 intersect.

L_2 is parallel to $\langle 1, 3, 2 \rangle$

$L_3 \parallel \parallel \langle -2, -6, -4 \rangle = -2 \langle 1, 3, 2 \rangle$. Hence L_2, L_3 are parallel. If we re-name the parameter in the equation of L_2 to s :

$L_2 : \quad x = 3 + s \quad y = 3s, \quad z = -3 + 2s$, and solve the system of equation

$$\begin{cases} 2t = 3 + s \\ 1 + t = 3s \end{cases}$$

we get $t = 2, s = 1$ is an answer, so L_1, L_2 intersect.